Where to put a comma

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Comma

I'll first formulate a few 'rules' to follow when using commas in an English text. The rules will then be illustrated by examples.

- Forget the rules of your native tongue.
- Better fewer commas than more.
- Put a comma to separate.
- Put a comma to indicate logical structure.
- Put the main comma, omit the secondary ones.
- Put a comma where the reader should make a pause.
- Do not follow blindly any rules, especially your own.
- Do not modify punctuation in proof-reading unless absolutely necessary.

Forget the rules of your own tongue

In many languages, commas are obligatory before specific words, but not in English:

No comma before 'that':

- We will prove that M is convex.

(Often) no comma before 'which/that':

- Every function which satisfies (2) is called a weak solution of equation (1).
- This holds for every map that appears in Table 2.

In both examples the relative clause ('which satisfies (2)' and 'that appears in Table 2') is *defining* (or 'restrictive') – you cannot remove it from the sentence. Defining relative clauses are *not* to be set off by commas.

However, if 'which' starts a non-restrictive clause (an additional comment), the commas should be used:

- The function u, which we have proved to be integrable, is a weak solution of (1).

(In general) no comma before 'if':

- A set is called *closed* if its complement is open.

(Often) there is a comma before 'and':

- A sketch proof is given in Section 6, and we refer the reader to [26] for more details about essential dimension.
- This conjecture was proved by Liebeck, and more recently it has been shown that $c = 7$ is the best possible constant.

We put a comma because the second part is long or starts a new thought.

Put a comma to separate

Put a comma to prevent 'false links':

- Thus the proof of Theorem 2, and of the remarks that follow it, is complete. [The commas prevent 'it is complete'.]
- Part (ii) follows immediately from Lemma 2, and part (iii) is an easy consequence of the fact that G has an orbit of dimension n. [The comma prevents 'follows from Lemma 2 and part (iii)'.]

Put a comma to separate symbols:

- For every $x \in X$, Y_x is complete.

Put commas to set off a complementary clause (one that can be omitted without changing the meaning):

- The *base size* of G, denoted by $b(G, H)$ (or just $b(G)$ if the context is clear), is the minimal size of a base for G.

Put a comma before a complementary clause ending a sentence:

- For example, Bochert's result was motivated by the classical problem of bounding the order of a finite primitive permutation group, which attracted a lot of attention in the 19th century.

Use commas to indicate logical structure

In logically complicated sentences, involving alternatives, conjunctions, implications and equivalences, the ranges of particular ‘operators’ may be unclear – then punctuation should help.

For example, if you write ‘Then either A or B and C’, it is not clear whether you mean

- Then either A or B holds, and moreover C is true,

or

- Then either A holds, or both B and C hold.

Also, do not write ‘Then A iff B and C’ if you mean ‘Then A iff B, and moreover C’. In the latter case (if you do not want to start a new sentence), use a semicolon: ‘Then A iff B; moreover, C’.

With very complicated structures, punctuation may not be enough; then a vertical (itemized) arrangement usually helps:

- Since either

  (i) A and $B(x)$ for some x, or

  (ii) D implies $C(x)$ for all x,

  it follows that

  (a) F,

  (b) G.

Put a comma where the reader should pause

- If not, then $K_j = K_j \cap H^g$ for all $g \in G$, which implies that K_j is a positive-dimensional normal subgroup.
- Indeed, the only known exceptions are the cases with H finite.
- A similar argument shows that $b \leq c$, hence equality holds.
- Determining the base size of a given permutation group is a classical problem in permutation group theory, with a long tradition and many applications.

No rules!

In many situations, you can use a comma or not. In each pair of examples below, both sentences have correct punctuation.

1. If A is compact then it is bounded.
2. If A is compact, then it is bounded.

1. In Section 5 we prove Theorem 3.
2. In Section 5, we prove Theorem 3.

1. For every x there exists y such that...
2. For every x, there exists y such that...

1. By (1) the set A is compact.
2. By (1), the set A is compact.

1. Clearly the set A is compact.
2. Clearly, the set A is compact.

However, if expressions like those above are parts of more complicated sentences, too much punctuation may be annoying or even confusing. For example, the sentence

- Since f is compact, by (1), the set A is bounded

is ambiguous: it is not clear which part the phrase 'by (1)' refers to; it is preferable to use one of the versions below:

1. Since f is compact by (1), the set A is bounded.
2. Since f is compact, by (1) the set A is bounded.

This shows that strict adherence to the rule 'I always put a comma after «by» phrases' might result in a mistake.

Similarly, even if you (can) put a comma after certain words when they appear at the beginning of a sentence, the comma is no more necessary in the middle:

1. Moreover, F is continuous.
2. Then F is continuous, and moreover we can approximate it by functions satisfying (5). [or without a comma before 'and']

1. Consequently, F is continuous.
2. Then F is continuous, and consequently we can approximate it by functions satisfying (5). [or without a comma before 'and']

Not just a comma between symbols

Sentences containing mathematical expressions separated only by commas are difficult to read, and their logical structure may be unclear. Replacing commas by words makes things clearer and spares the reader's effort.

For example:

- Then $f(x) > 0$, $x \in X$.

At least formally, one can understand it in several ways:

- Then $f(x) > 0$ and $x \in X$.
- Then $f(x) > 0$ for all $x \in X$.
- Then $f(x) > 0$ for some $x \in X$.

Moreover, remember a popular convention: a displayed formula

$$f(x) > 0, \qquad x \in X,$$

will be understood as '$f(x) > 0$, $\forall x \in X$' by most readers. If you mean $\exists$, write

$$\exists x \in X, \quad f(x) > 0.$$

Not just a comma between separate sentences

Do not put just a comma between separate sentences (where a full stop or a semicolon would be possible).

Instead of 'Let $x \in X$, then $f(x)$ is convex', write

- Let $x \in X$. Then $f(x)$ is convex.

Instead of 'The first inequality is well known, see [5, Proposition 3.1]' you can use one of the following:

- The first inequality is well known: see [5, Proposition 3.1].
- The first inequality is well known (see [5, Proposition 3.1]).
- The first inequality is well known – see [5, Proposition 3.1].
- The first inequality is well known; see [5, Proposition 3.1].
- The first inequality is well known [5, Proposition 3.1].

In the last example, you need not add ( ): the square brackets are enough.

Commas in lists

You can write either

- Cox, Brown and Fox

or

- Cox, Brown, and Fox

(various manuals recommend one of these versions, but there is no general agreement, neither in British nor in American publications). Adding a comma is always recommended if it helps to avoid ambiguity.

Semicolon

Sometimes two sentences are independent, but so closely connected that you do not want to separate them by a full stop. You can then use a semicolon:

- Using Theorem 6 we can compute the precise value of $A(v)$ in almost all cases; this is a significant strengthening of the general estimate $A(v) \leq 2$ stated in [6].
- A subset of Ω is a *base* for G if its pointwise stabilizer is trivial; the *base size* of G is the minimal cardinality of a base.
- Then $f_1 = g_1$ in D_1; and since D_2 is connected, we also have $f_1 = g_1$ in D_2.
- We will only prove (i); the proof of (ii) is similar.
- The first inequality is well known; see [5, Proposition 3.1] for a similar statement.

Use semicolons between items of a list when the items are long and complex or involve commas:

- Here A, B and C are complex numbers depending on x, y and z; D and E depend on c and d; and F is given by Lemma 5.

Colon

You can put a colon between two sentences if the second explains, illustrates, paraphrases or gives more detail about the first:

- Here is a more explicit statement of what the theorem asserts: if γ can be continued analytically along C_1 to f_1, and if it can also be continued analytically along C_2 to f_2, then $f_1 = f_2$ in $C_1 \cap C_2$.
- A few S-invariant subspaces are immediately apparent: if Y_k is the set of all x whose first k coordinates are 0, then Y_k is S-invariant.

If the explanation after the colon involves more than one sentence, start it with a capital letter:

- Actually, H^p is a Banach space: To prove the completeness, let (f_n) be a Cauchy sequence in H^p. Then...

Do *not* put a colon before every displayed formula.

In English, a colon rarely appears in the middle of a sentence. In particular, do *not* put a colon before a list in the middle of a sentence:

- The group G can be of type A, B, C or D.
- Consider all elements $x \in X$ such that (i) ..., (ii) ..., and (iii) ...

However, put a colon before a list when the sentence preceding the list is complete (e.g. contains 'the following' or 'as follows'):

- The group G can be of four types: A, B, C or D.
- Consider all elements $x \in X$ such that one of the following holds:

  (i) ...;
  (ii) ...

(See also 'Vertical lists'.)

Vertical lists

If a vertical list begins in the middle of sentence (that is, it is not preceded by a complete sentence), then normally we do not put a colon before the list, and the items begin with a lower case letter and end with a comma or a semicolon:

- Let A be the set of all $x \in B$ such that

  (i) the function f_x is discontinuous;
  (ii) the function g_x is continuous, but not strictly increasing.

However, sometimes the items of a list contain full stops, or you may wish to end each item with a full stop. Then put a colon before the list and start each item with a capital letter:

- The following conditions are equivalent:

  (i) The mapping F is convex.
  (ii) The manifold M is simply connected.

- Then:

  (i) F is continuous, convex and bounded from below. Moreover, there is no G satisfying (4) for $A = F$.
  (ii) F' is continuous and bounded from below, but not convex. However, there is a G satisfying (4) for $A = F'$.

Dash

The dash is used in English in a different way than in some other languages.

Sudden break:

- Recall the interior tensor product $A \otimes B$ – this is a right module over C.
- The first inequality is well known – see [5, Proposition 3.1].

In print, the dash may also be set in another way:

- Recall the interior tensor product $A \otimes B$—this is a right module over C.
- The first inequality is well known—see [5, Proposition 3.1].

The dash is not used to avoid repetitions:

Do not use a dash to replace a word you do not want to repeat:

- We denote the former set by A, and the latter by B. [not 'the latter – by B']

Avoid using dashes next to mathematical expressions, which may be confusing.

Hyphen

In the spelling of many words, you can use a hyphen or not, but be consistent within each document:

Non(-):

You can write either

- nontrivial, nonzero, nonnegative,

or

- non-trivial, non-zero, non-negative.

However, always write

- non-Euclidean, non-locally-convex [better than 'non-locally convex' because what you are negating here is not just 'locally'].

Hyphen mandatory:

- one-parameter group

- one-to-one map
- two-stage computation
- n-fold integration

The hyphen is useful (and necessary) when e.g. you want to make several words joined by prepositions and/or conjunctions into one 'qualifier':

- a local-to-global method, an easy-to-trace element

Hyphen optional:

- right hand side, right-hand side
- second order equation, second-order equation
- selfadjoint, self-adjoint
- seminorm, semi-norm
- a blow-up, a blow up, a blowup [but always 'to blow up' (a verb)]
- the nth element, the n-th element
- above-mentioned, above mentioned
- well-known, well-defined, well known, well defined

A general remark: nowadays, hyphens tend to be less and less used: the expressions that used to be hyphenated are now often written as one word without a hyphen or as two separate words.
Personally, I'd always write 'a finite-dimensional space', 'a model-theoretic construction', but these expressions are now very often written as separate words.

Hyphens and mathematics:

Write 'G-space' [code: `$G$-space`], and not '$G-$space'.
Also, an operator may be 'L^p-L^q continuous', coded

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$L^p$-$L^q$ continuous,
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but not '$L^p - L^q$ continuous'.

N-rule:

In several cases, a double hyphen is used ('N-rule'):

- pages 12–34, items (i)–(iv), Sections 2.3–2.7 [to indicate ranges]
- Radon–Nikodym theorem [for two names; but 'Piatetski-Shapiro' because this is one name]