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Chernoff approximation of evolution semigroups and beyond.

We present a method to approximate evolution semigroups with the help of the Chernoff theorem. We discuss different approaches to construct Chernoff approximations for Feller semigroups and Schrödinger groups. In particular, we outline the techniques based on pseudo-differential operators, shifts, rotation. We show how to approximate semigroups obtained from some original (known or already approximated) ones by such procedures as additive and/or multiplicative perturbations of generators, subordination, adding Dirichlet boundary/external conditions ($\sim$ killing of underlying stochastic processes). The described approaches allow to approximate semigroups generated, e.g., by subordinate Feller diffusions on star graphs and Riemannian manifolds. Moreover, the constructed Chernoff approximations for evolution semigroups can be used further to approximate solutions of some time-fractional evolution equations describing anomalous diffusion (solutions of such equations do not posess the semigroup property).

Many Chernoff approximations lead to representations of solutions of (corresponding) evolution equations in the form of limits of n-fold iterated integrals of elementary functions when n tends to infinity. Such representations are called *Feynman formulae*. They can be used for direct computations, modelling of the related dynamics, simulation of stochastic processes. Furthermore, the limits in Feynman formulae sometimes coincide with path integrals with respect to probability measures (such path integrals are usually called *Feynman-Kac formulae*) or with respect to Feynman type pseudomeasures (such integrals are *Feynman path integrals*). Therefore, the constructed Feynman formulae can be used to approximate (or even sometimes to define) the corresponding path integrals; different Feynman formulae for the same semigroup allow to establish connections between different path integrals. Moreover, in some cases, Feynman formulae provide Euler–Maruyama schemes for SDEs; some Chernoff approximations can be understood as a version of the operator splitting method (known in the numerics of PDEs).

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