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Implicit Fokker-Planck Equations: Non-commutative Convolution of Probability Distributions

Feller's operator representation of a single homogenous Markov transition function intertwined by the Chapman Kolmogorov equation is by virtue of the Feller convolution. This convolution is a single state construct and fails to deal with a pair of homogenous Markov transition functions intertwined by an extended Chapman Kolmogorov equation with distinct state spaces. Extended Chapman Kolmogorov equation with distinct state spaces naturally arise in the pair of pseudo Poisson processes of [2, §2] where the distinct state spaces were discrete. The distinctness of the state spaces renders Feller's notion of a simple conditional distribution (mixture) [1, Definition (9.3), V.9] inadequate in describing such pairs. Therefore we introduce a new type of stochastic kernel.

This talk introduces a non-commutative generalization of the Feller convolution in the form of the product of a convolution algebra of admissible homomorphisms to deal with Markov transition functions that arise in the setting of distinct state spaces. This new framework of 'quaternion'-valued admissible homomorphisms on a product test space (i) provides a mathematical interpretation of what Feller created by intuition and (ii) shows that the Fokker-Planck equation for distinct two-state space homogenous processes intertwined by an extended Chapman-Kolmogorov equation is a particular case of an implicit B-evolution equation of empathy theory. Thus, the framework of admissible homomorphisms extends the classical framework of probability distributions.

References

[1] W. Feller, *An introduction to probability theory and its applications, vol ii, second edition*, Wiley Series in Probability and Mathematical Statistics, 1971.

[2] W. Lee and N. Sauer, *Intertwined markov processes: the extended chapman-kolmogorov equation*, Proc. R. Soc. Edinb. A (2017).