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From traveling and standing fronts on the curved surfaces to pattern formation

We analyze heteroclinic traveling waves propagating on two dimensional manifolds to show that the modification of the inherent front velocity, in the limit of diffusion tending to zero, is proportional to the geodesic curvature of the front line. As a result, on concave surfaces, stable standing fronts can be formed on lines of constant geodesic curvature. These lines minimize the geometric functional describing the system's energy, consisting of terms proportional to front line-length and to the inclosed surface area. Front pinning at portions of surface with the negative Gaussian curvature provides a mechanism of pattern formation that connects intrinsic surface geometry with the arising pattern, previously characterized by us in 3D case [1]. By considering a system of equations modeling boundary-volume interaction, we show that polarization of the boundary may induce polarization inside the volume. Finally, we provide a link between dynamics of traveling fronts and quantum vortices in superfluids [2], allowing to demonstrate existence of three families of self-similar solutions for front line motion.

References

[1] S. Bialecki, B. Kazmierczak, and T. Lipniacki, *Polarization of concave domains by traveling wave pinning*, PLOS ONE **12** (2017), e0190372.

[2] T. Lipniacki, *Shape-preserving solutions for quantum vortex motion under localized induction approximation*, Physics of Fluids **15** (2003), 1381–1395.