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Generators, martingale problems, and stochastic equations

A natural way of specifying a Markov process is by defining its generator. Classically, one then shows that the generator, or some natural extension, is the generator of a positive, contraction semigroup which determines the transition function of the Markov process which must satisfy the Kolmogorov *forward* and *backward* equations.

Standard semigroup identities and the relationship between the process and the semigroup also imply that the process has certain martingale properties which are the basis for the classical identity known as Dynkin's identity. In work on diffusions, Stroock and Varadhan, exploiting these properties, formulated a *martingale problem* as an approach to uniquely determining the process corresponding to the generator.

For many processes, in particular diffusions, the process can also be determined as a solution of a stochastic equation. Very generally, the forward equation, the martingale problem, and, if one exists, the corresponding stochastic equation, are equivalent in the sense that a solution of one corresponds to solutions of the others. In particular, uniqueness of one implies uniqueness of the other.

The formulation of the three problems becomes more complicated in the case of constrained Markov processes (for example, reflecting diffusions). Associating a constrained martingale problem with a certain controlled martingale problem in a sense reduces the problem of equivalence of the three approaches to specifying the process to the unconstrained case. The forward equations, martingale problems, and (in examples) stochastic equations will be formulated and proof of their equivalence outlined.