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Time asymptotics of structured populations with diffusion

In this talk we will study the following size-structured population model [1]

$$\begin{aligned}\partial_t u(t,s) + \partial_s(\gamma(s)u(t,s)) = \partial_s(d(s)\partial_s(u(t,s))) \quad &- \quad \mu(s)u(t,s) \\ &+ \quad \int_0^m \beta(s,y)u(t,y)\mathrm{d}y,\end{aligned}$$

for $s \in [0,m]$ and $m < \infty$. We consider some Feller boundary conditions and we verify that the problem is well-posed in the sense of the semigroups theory in $L^1(0,m) \times \mathbb{R}^2$. With Hopf maximum principle we prove that the semigroup is irreducible. Using weak compactness arguments, we show first a stability result of the essential type then deduce that the semigroup has a spectral gap and consequently the asynchronous exponential growth property. Finally [2] we show how to extend this theory to models with arbitrary sizes and give a theoretical condition to get the asynchronous behavior.

References

[1] J.Z. Farkas and P. Hinow, *Physiologically structured populations with diffusion and dynamic boundary conditions*, Mathematical Biosciences and Engineering (2011), 503–513.

[2] M. Mokhtar-Kharroubi and Q. Richard, *Time asymptotics of structured populations with diffusion and dynamic boundary conditions*, to appear in Discrete and Continuous Dynamical Systems - series B.