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On the Harmonic Extension Approach to Fractional Powers of Linear Operators

In [2] the authors attracted a lot of attention when describing the action of the fractional Laplacian $(-\Delta)^\alpha$, $\alpha \in (0,1)$, on $f \in L^2(\mathbb{R}^n)$ using a solution u of the ODE

$$u'' + \frac{1-2\alpha}{t}u' = -\Delta u$$

in $L^2(\mathbb{R}^n)$. One has $u(0) = f$ which is why one can consider u as an extension of f and especially in the case of the Laplacian one may interpret u as a harmonic function in "$\mathbb{R}^{n+2-2\alpha}$". Until now many other authors ([1, 3, 4]) contributed by discussing the approach for general sectorial operators A instead of $-\Delta$. Still it is an open problem whether the approach works for all sectorial operators and whether the above introduced solution is actually unique. The talk aims for giving some partial results on these questions.

References

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[3] J. E. Galé, P. J. Miana, and P. R. Stinga, *Extension problem and fractional operators: semigroups and wave equations*, Journal of Evolution Equations **13** (2013), no. 2, 343–368.

[4] P. R. Stinga and J. L. Torrea, *Extension Problem and Harnack's Inequality for Some Fractional Operators*, Comm. Partial Differential Equations **35** (2010), no. 11, 2092–2122.