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Elliptic operators with unbounded diffusion, drift and potential terms

We prove that the realization A_p in $L^p(\mathbb{R}^N)$, $1 < p < \infty$, of the elliptic operator $A = (1+|x|^\alpha)\Delta + b|x|^{\alpha-1}\frac{x}{|x|}\cdot\nabla - c|x|^\beta$ with domain $D(A_p) = \{u \in W^{2,p}(\mathbb{R}^N) \mid Au \in L^p(\mathbb{R}^N)\}$ generates a strongly continuous analytic semigroup $T(\cdot)$ provided that $\alpha > 2$, $\beta > \alpha - 2$ and any constants $b \in \mathbb{R}$ and $c > 0$. This generalizes the recent results in [1] and in [2].

Moreover we prove that the heat kernel k associated to A satisfies

$$k(t,x,y) \le c_1 e^{\lambda_0 t + c_2 t^{-\gamma}} \left(\frac{1+|y|^\alpha}{1+|x|^\alpha}\right)^{\frac{b}{2\alpha}} \frac{(|x||y|)^{-\frac{N-1}{2}-\frac{1}{4}(\beta-\alpha)}}{1+|y|^\alpha} e^{-\frac{\sqrt{2}}{\beta-\alpha+2}\left(|x|^{\frac{\beta-\alpha+2}{2}}+|y|^{\frac{\beta-\alpha+2}{2}}\right)}$$

for $t > 0$, $|x|, |y| \ge 1$, where $b \in \mathbb{R}$, c_1, c_2 are positive constants, λ_0 is the largest eigenvalue of the operator A, and $\gamma = \frac{\beta-\alpha+2}{\beta+\alpha-2}$. The proof is based on the relationship between the log-Sobolev inequality and the ultracontractivity of a suitable semigroup in a weighted space.

Joint work with S.E. Boutiah, F. Gregorio and A. Rhandi.

References

[1] A. Canale, A. Rhandi, and C. Tacelli, *Schrödinger type operators with unbounded diffusion and potential terms*, Ann. Sc. Norm. Super. Pisa Cl. Sci. **XVI** (2016), no. 2, 581–601.

[2] G. Metafune, C. Spina, and C. Tacelli, *Elliptic operators with unbounded diffusion and drift coefficients in L^p spaces*, Adv. Diff. Equat **19** (2012), no. 5-6, 473–526.