Semigroups of Operators: Theory and Applications

Book of abstracts

Kazimierz, Poland, September 30— October 5, 2018

Contents

Conference schedule

Sunday, September 30

14^{00} Check in and registration

18^{00} Dinner

Monday, October 1

7^{45} Breakfast

8^{55} Conference opening

9^{00} Plenary talk: Charles Batty

10^{00} Coffee break

10^{20} Morning sessions

	Y. Tomilov		J. Banasiak and A. Bobrowski
10^{20}–10^{45}	David Seifert	10^{20}–10^{45}	Wilson Lamb
10^{50}–11^{15}	Jan Rozendaal	10^{50}–11^{15}	Lyndsay Kerr
11^{20}–11^{45}	Lassi Paunonen	11^{20}–11^{45}	Luke Joel
11^{50}–12^{15}	Filippo Dell'Oro	11^{50}–12^{15}	Mustapha Mokhtar-Kharroubi
12^{20}–12^{45}	Jochen Glück	12^{20}–12^{45}	Roksana Brodnicka
12^{50}–13^{15}	Luciano Abadias	12^{50}–13^{15}	Quentin Richard

13^{30} Lunch

15^{00} Afternoon sessions (part 1):

	Y. Tomilov		J. Voigt
15^{00}–15^{25}	Logan Hart	15^{00}–15^{25}	Marjeta Kramar-Fijavž
15^{30}–15^{55}	Marcin Moszyński	15^{30}–15^{55}	Damir Kinzebulatov

16^{00} Coffee break

16^{30} Afternoon sessions (part 2):

	A. Bobrowski		J. Voigt
16^{30}–16^{55}	Jerzy Zabczyk	16^{30}–16^{55}	Christian Budde
17^{00}–17^{25}	Wojciech Kryszewski	17^{00}–17^{25}	Abed Boulouz
17^{30}–17^{55}	Grzegorz Łukaszewicz	17^{30}–17^{55}	Sascha Trostorff

18^{30} Dinner (barbecue)

Tuesday, October 2

7^{45} Breakfast

9^{00} Plenary talk: Jan Kisyński

10^{00} Plenary talk: Jerry Goldstein

11^{00} Coffee break

11^{30} Morning sessions

	G. Sviridyuk		D. Mugnolo and M. Kramar–Fijavž
11^{30}–12^{05}	Georgy Sviridyuk	11^{30}–11^{55}	Serge Nicaise
12^{10}–12^{30}	Alyona Zamyshlyaeva	12^{00}–12^{25}	Amru Hussein
12^{35}–12^{55}	Alevtina Keller	12^{30}–12^{55}	Talk cancelled

13^{15} Lunch

14^{30} Afternoon sessions (part 1):

	G. Sviridyuk		D. Mugnolo and M. Kramar–Fijavž
14^{30}–14^{55}	Natalia Manakova	14^{30}–14^{55}	Federica Gregorio
15^{00}–15^{25}	Evgeniy Bychkov	15^{00}–15^{25}	Christian Seifert
15^{30}–15^{55}	Natalya Solovyova	15^{30}–15^{55}	James Kennedy

16^{00} Coffee break

16^{30} Afternoon sessions (part 2):

	A. Bobrowski		J. Voigt
16^{30}–16^{55}	Abdelaziz Rhandi	16^{30}–16^{55}	Mustapha Mokhtar-Kharroubi
17^{00}–17^{25}	Michael Kaplin	17^{00}–17^{25}	Sergey Piskarev
17^{30}–17^{55}	Jacek Polewczak	17^{30}–17^{55}	Jan Meichsner

18^{15} Dinner

Wednesday, October 3

7^{15} Breakfast

8^{15} Tour to Kozlowka Palace, 10^{15} Guided tour in Kazimierz

14^{00} Lunch

15^{00} Afternoon sessions (part 1):

	A. Bobrowski		M. Lachowicz and M. Rosini
15^{00}–15^{25}	Gisèle Goldstein	15^{00}–15^{25}	Jurij Kozicki
15^{30}–15^{55}	Tim Binz	15^{30}–15^{55}	Waldemar Cieślak

16^{00} Coffee break

16^{30} Afternoon sessions (part 2):

	J. Banasiak and A. Bobrowski		M. Lachowicz and M. Rosini
16^{30}–16^{55}	Andrzej Komisarski	16^{30}–16^{55}	Viktor Gerasimenko
17^{00}–17^{25}	Bogdan Kaźmierczak	17^{00}–17^{25}	Henryk Leszczyński
17^{30}–17^{55}	Tomasz Lipniacki	17^{30}–17^{55}	Massimiliano Rosini

18^{30} Dinner

Thursday, October 4

7^{45} Breakfast

9^{00} Plenary talk: Thomas G. Kurtz

10^{00} Coffee break

10^{30} Morning sessions

	K. Bogdan		A. Rhandi
10^{30}–10^{55}	Agnieszka Kałamajska	10^{30}–10^{55}	Wolfgang Ruess
11^{00}–11^{25}	Artur Rutkowski	11^{00}–11^{25}	Luca Lorenzi
11^{30}–11^{55}	Victoria Knopova	11^{30}–11^{55}	Luciana Angiuli
12^{00}–12^{25}	Tomasz Jakubowski	12^{00}–12^{25}	Monika Wrzosek
12^{30}–12^{55}	Yana Butko	12^{30}–12^{55}	Delio Mugnolo

13^{00} Group photo

13^{30} Lunch

14^{30} Afternoon sessions (part 1):

	K. Bogdan		A. Rhandi
14^{30}–14^{55}	Krzysztof Bogdan	14^{30}–14^{55}	Talk cancelled
15^{00}–15^{25}	Łukasz Leżaj	15^{00}–15^{25}	Chiara Spina
15^{30}–15^{55}	Karol Szczypkowski	15^{30}–15^{55}	Cristian Tacelli

16^{00} Coffee break

16^{30} Afternoon sessions (part 2):

	A. Bobrowski		R. Rudnicki
16^{30}–16^{55}	Stanisław Kwapień	16^{30}–16^{55}	Aleksandra Puchalska
17^{00}–17^{25}	Wha-Suck Lee	17^{00}–17^{25}	Henrik Kreidler
17^{30}–17^{55}	Ami Viselter	17^{30}–17^{55}	Paweł Klimasara

18^{30} Concert of Chamber Music

19^{30} Conference Dinner

Friday, October 5

7^{45} Breakfast

9^{00} Plenary talk: Roberto Triggiani

10^{00} Coffee break

10^{30} Morning sessions

	J. Voigt		R. Rudnicki
10^{30}–10^{55}	Hendrik Vogt	10^{30}–10^{55}	Marta Tyran-Kamińska
11^{00}–11^{25}	Josef Kreulich	11^{00}–11^{25}	Radosław Wieczorek
11^{30}–11^{55}	Markus Kunze	11^{30}–11^{55}	Andrzej Tomski
12^{00}–12^{25}	Christian Seifert	12^{00}–12^{25}	Adam Gregosiewicz
12^{30}–12^{55}	Jürgen Voigt	12^{30}–12^{55}	Ryszard Rudnicki

13^{00} Conference closing

13^{15} Farewell lunch

14^{00} – 15^{00} Buses to Warsaw.

Sessions

Plenary talks

1. Charles Batty, Functional calculus for analytic Besov functions.
2. Jerry Goldstein, The Agmon-Douglis-Nirenberg Problem for Dynamic Boundary Conditions.
3. Jan Kisyński, Petrovsky condition for forward evolution and semigroups.
4. Thomas G. Kurtz, Generators, martingale problems, and stochastic equations.
5. Roberto Triggiani, A third order (in time) PDE: a view from the boundary, to control and to observe.

1. Nonlocal operators (K. Bogdan)

1. Krzysztof Bogdan, Heat kernel of isotropic nonlocal operators.
2. Yana Butko (Kinderknecht), Chernoff approximation of evolution semigroups and beyond.
3. Tomasz Jakubowski, Critical negative Schrödinger perturbations of fractional Laplacian.
4. Agnieszka Kałamajska, Dirichlet's problem for critical Hamilton-Jacobi fractional equation.
5. Victoria Knopova, Long-time behaviour of some Markov processes.
6. Łukasz Leżaj, Heat kernels for subordinators.
7. Artur Rutkowski, The Dirichlet problem for nonlocal Lévy-type operators.
8. Karol Szczypkowski, Fundamental solution for super-critical non-symmetric Lévy-type operators.

2. Evolution equations of biosciences and bioengineering (J. Banasiak, A. Bobrowski)

1. Roksana Brodnicka, Asymptotic stability of an evolutionary nonlinear Boltzmann-type equations.
2. Luke Joel, The Discrete Unbounded Coagulation-Fragmentation Equation with Growth, Decay and Sedimentation.

3. Bogdan Kaźmierczak, Existence of solutions in a model of bone pattern formation.
4. Lyndsay Kerr, Discrete Coagulation-Fragmentation System.
5. Andrzej Komisarski, On a model of the ideal heat exchanger and its relation to the telegrapher's equations.
6. Wilson Lamb, Discrete Fragmentation Equations.
7. Tomasz Lipniacki, From traveling and standing fronts on the curved surfaces to pattern formation.
8. Mustapha Mokhtar-Kharroubi, Existence of invariant densities for conservative linear kinetic equations on the torus without spectral gaps.
9. Quentin Richard, Time asymptotics of structured populations with diffusion.

3. Multiscale Approaches and the Semigroup Environments (M. Lachowicz, M. D. Rosini)

1. Waldemar Cieślak, The Fuss formulas in the Poncelet porism.
2. Viktor Gerasimenko, On Semigroups of Operators Describing Processes of Creation and Propagation of Quantum Correlations.
3. Jurij Kozicki, Infinite populations of interacting entities as complex systems: multiscale Markov dynamics.
4. Henryk Leszczyński, Self-organization with small range interactions: Creation of bipolarity.
5. Massimiliano D. Rosini, On the micro-to-macro limit for 1D scalar conservation laws.

4. Semigroups on networks and further ramified structures (D. Mugnolo, M. Kramar-Fijavž)

1. Federica Gregorio, Bi-Laplacians on graphs and networks.
2. Amru Hussein, Non-self-adjoint graphs.
3. James Kennedy, Hot spots of quantum graphs.
4. Serge Nicaise, Dispersive effects for the Schrödinger equation on graphs.
5. Christian Seifert, The linearized KdV equation on metric graphs.

5. Semigroups for parabolic problems (A. Rhandi)

1. Luciana Angiuli, On systems of parabolic equations with unbounded coefficients (Part II).
2. Luca Lorenzi, On systems of parabolic equations with unbounded coefficients (Part I).
3. Delio Mugnolo, Comparison principles for parabolic equations and applications to PDEs on networks.
4. Wolfgang Ruess, Regularity of solutions to partial differential delay equations.
5. Chiara Spina, Rellich and Calderón-Zygmund inequalities for operators with discontinuous and singular coefficients.
6. Cristian Tacelli, Elliptic operators with unbounded diffusion, drift and potential terms.
7. Monika Wrzosek, Newton's method for the McKendrick equation.

6. Stochastic semigroups and their applications in biology (R. Rudnicki)

1. Adam Gregosiewicz, Asymptotics of the Lebowitz–Rubinow–Rotenberg model of cell populations development.
2. Paweł Klimasara, A model for random fire induced tree-grass coexistence in savannas.
3. Henrik Kreidler, Weighted Koopman semigroups and their applications.
4. Aleksandra Puchalska, The graph structure impact on a singular limit of the generalized network transport.
5. Ryszard Rudnicki, Stochastic semigroups and their applications to Stein's neural model.
6. Andrzej Tomski, Semigroups in biophysics: stochastic Liouville equation.
7. Marta Tyran-Kamińska, Substochastic semigroups and positive perturbations of boundary conditions.
8. Radosław Wieczorek, Clustering in a model of yeast cell cycle.

7. Quantitative aspects of semigroup asymptotics (Y. Tomilov)

1. Luciano Abadias, Mean ergodic theorems and domains of higher degree functions of Cesàro bounded operators.
2. Filippo Dell'Oro, Decay properties of dissipative systems of linear thermoelasticity and viscoelasticity.
3. Jochen Glück, Convergence of Positive Semigroups and Hyper-Bounded Operators.
4. Logan Hart, Superstability of Semigroups.
5. Marcin Moszyński, Uni-asymptotic linear systems.
6. Lassi Paunonen, Nonuniform Stability Properties of Coupled Systems.
7. Jan Rozendaal, Sharp growth rates for semigroups using resolvent bounds.
8. David Seifert, Optimal rates of decay for semigroups on Hilbert spaces.

8. Perturbation and Approximation (J. Voigt)

1. Abed Boulouz, On norm continuity, differentiability and compactness of perturbed semigroups.
2. Christian Budde, Extrapolation spaces and Desch-Schappacher perturbations of bi-continuous semigroups.
3. Marjeta Kramar-Fijavž, On perturbing the domain of certain generators.
4. Damir Kinzebulatov, A new approach to the L^p theory of $-\Delta+b\cdot\nabla$ and its application to Feller processes with general drifts.
5. Josef Kreulich, On compactifications of bounded C_0-semigroups.
6. Markus Kunze, Diffusion with nonlocal Dirichlet boundary conditions on unbounded domains.
7. Jan Meichsner, On the Harmonic Extension Approach to Fractional Powers of Linear Operators.
8. Mustapha Mokhtar-Kharroubi, Relative operator bounds for positive operators in ordered Banach spaces and related topics.
9. Sergey Piskarev, Approximation of fractional differential equations in Banach spaces.

10. Christian Seifert, Perturbations of positive semigroups with applications to Dirichlet forms perturbed by jump parts.
11. Sascha Trostorff, Strongly continuous semigroups associated with evolutionary equations.
12. Hendrik Vogt, Perturbation theory for accretive operators in L_p.
13. Jürgen Voigt, On "monotone" convergence of sectorial forms.

9. Varia (A. Bobrowski)

1. Tim Binz, Operators with Wentzell boundary conditions and the Dirichlet-to-Neumann operator.
2. Gisèle Goldstein, New Results on Instantaneous Blowup in $\mathbb{H}^N$.
3. Michael Kaplin, Relatively uniformly continuous semigroups on vector lattices.
4. Wojciech Kryszewski, Bifurcation at infinity for elliptic problems on $\mathbb{R}^N$.
5. Stanisław Kwapień, Continuity and boundness of stochastic convolutions.
6. Wha-Suck Lee, Implicit Fokker-Planck Equations: Non-commutative Convolution of Probability Distributions.
7. Grzegorz Łukaszewicz, Nonlinear semigroups in hydrodynamics and their perturbations. Micropolar meets Newtonian.
8. Jacek Polewczak, Hard-spheres linear kinetic theories.
9. Abdelaziz Rhandi, Weighted Hardy's inequalities and Kolmogorov-type operators.
10. Ami Viselter, Convolution semigroups on quantum groups and non-commutative Dirichlet forms.
11. Jerzy Zabczyk, Markovian models of short rates.

10. Sobolev type equations. Degenerate operator semigroups and propagators (G. Sviridyuk)

1. Evgeniy Bychkov, Semilinear Sobolev Type Equations of Higher Order.
2. Georgy Sviridyuk, Sobolev type equations. Degenerate semigroups of operators and degenerate propagators.

3. Alevtina Keller, Algorithms for the Numerical Solution of Optimal Control Problems for Models of the Leontief Type.
4. Natalia Manakova, Optimal Control Problem for the Sobolev Type Equations.
5. Natalya Solovyova, Nonclassical conditions for linear Sobolev Type Equations.
6. Alyona Zamyshlyaeva, Sobolev type equations of higher order. Theory and applications.

Luciano Abadias
University of Zaragoza, Spain

Mean ergodic theorems and domains of higher degree functions of Cesàro bounded operators

In this talk I will present mean ergodic results for Cesàro bounded operators T on Banach spaces X, improving some of them existing for power bounded operators. In particular, we prove that for power bounded operators, T is (C,β)-ergodic for some $\beta > 0$ if and only if $X = \mathrm{Ker}(I-T) \oplus \overline{\mathrm{Ran}(I-T)}$, where (C,β)-ergodicity implies the classical mean ergocicity for $0 < \beta < 1$. The relations between the rates of convergence in mean ergodic results and the solutions to equations $(I-T)^s x = y$, $s \in (0,1]$, lead to discuss the domains of certain class of functions of Cesàro bounded operators T, in particular, the domain of $(I-T)^{-s}$, $s \in (0,1)$, and the domain of $\log(I-T)$ (infinitesimal generator of the holomorphic semigroup $((I-T)^s)_{\Re s>0}$). To prove the cited results, we use functional calculus on fractional Wiener algebras induced by this class of operators, and properties of regularity of holomorphic functions on the unit disc.

This is joint work with José E. Galé and Carlos Lizama.

Luciana Angiuli
University of Salento, Italy

On systems of parabolic equations with unbounded coefficients (Part II)

I will continue the investigation on systems of parabolic equations (coupled up to the first order) with unbounded coefficients defined in the whole $\mathbb{R}^d$, started in the talk by L. Lorenzi. In particular I will consider the case when the problem is set in an L^p-context more appropriate than the Lebesgue one. After giving the definition of *systems of invariant measures* which extends to the vector-valued case the notion of invariant measure of the scalar case, I will show some properties of the solution of the Cauchy problem associated, in these L^p-spaces.

Charles Batty
University of Oxford, United Kingdom

Functional calculus for analytic Besov functions

I shall describe work with Alexander Gomilko and Yuri Tomilov in which we develop a bounded functional calculus for analytic Besov functions applicable to the generators of many bounded semigroups, including bounded semigroups on Hilbert spaces and bounded holomorphic semigroups on Banach spaces. The calculus is a natural extension of the classical Hille-Phillips functional calculus, and it is compatible with the other well-known functional calculi. It satisfies the standard properties of functional calculi, provides a unified and direct approach to a number of norm-estimates in the literature, and enables improvements of some of them.

Tim Binz
Tübingen University, Germany

Operators with Wentzell boundary conditions and the Dirichlet-to-Neumann operator

We relate the generator property of an operator A with (abstract) generalized Wentzell boundary conditions on a Banach space X and its associated (abstract) Dirichlet-to-Neumann operator N acting on a "boundary" space ∂X. Our approach is based on similarity transformations and perturbation arguments and allows to split A into an operator A_{00} with Dirichlet-type boundary conditions on a space X_0 of states having "zero trace" and the operator N. If A_{00} generates an analytic semigroup, we obtain under a weak Hille–Yosida type condition that A generates an analytic semigroup on X if and only if N does so on ∂X. Here we assume that the (abstract) "trace" operator $L : X \to \partial X$ is bounded what is typically satisfied if X is a space of continuous functions. Concrete applications are made to various second order differential operators.

Krzysztof Bogdan
Wrocław University of Technology, Poland

Heat kernel of isotropic nonlocal operators

I will discuss joint results with Tomasz Grzywny and Michał Ryznar [1, 3, 2]. Let $d = 1, 2, \ldots$. Let $\nu\colon [0,\infty) \to (0,\infty]$ be nonincreasing and denote $\nu(z) = \nu(|z|)$ for $z \in \mathbb{R}^d$. In particular, $\nu(z) = \nu(-z)$. We assume that $\int_{\mathbb{R}^d} \nu(z)dz = \infty$ and

$$\int_{\mathbb{R}^d} \left(|z|^2 \wedge 1\right) \nu(z)dz < \infty.$$

Summarizing, ν is a strictly positive density function of an isotropic infinite unimodal Lévy measure on $\mathbb{R}^d$. In short – ν is *unimodal*. For $x \in \mathbb{R}^d$ and $u : \mathbb{R}^d \to \mathbb{R}$,

$$\begin{aligned} Lu(x) &:= \lim_{\epsilon\to 0^+} \int_{|x-y|>\epsilon} (u(y) - u(x))\nu(y - x)\, dy \\ &= \tfrac{1}{2}\int_{\mathbb{R}^d} (u(x + z) + u(x - z) - 2u(x))\nu(z)\, dz. \end{aligned}$$

The limit exists, e.g., for $u \in C_c^\infty(\mathbb{R}^d)$, the smooth functions with compact support. Note that L is a non-local symmetric translation-invariant linear operator on $C_c^\infty(\mathbb{R}^d)$ with positive maximum principle. For example, if $0 < \alpha < 2$ and

$$\nu(z) = \frac{2^\alpha \Gamma((d+\alpha)/2)}{\pi^{d/2}|\Gamma(-\alpha/2)|}|z|^{-d-\alpha}, \quad z \in \mathbb{R}^d,$$

then L is the fractional Laplacian, denoted by $\Delta^{\alpha/2}$. In general, L is the generator of a Markovian semigroup–the transition semigroup of the isotropic pure-jump Lévy processes $\{X_t, t \geq 0\}$ with the Lévy measure ν. Thus, X is a càdlàg stochastic process with law $\mathbb{P}$, such that $X(0) = 0$ almost surely, the increments of X are independent, with radially nonincreasing density function $p_t(x)$ on $\mathbb{R}^d$, and the following Lévy-Khintchine formula holds for $\xi \in \mathbb{R}^d$:

$$\mathbb{E}e^{i\langle\xi,X_t\rangle} = \int_{\mathbb{R}^d} e^{i\langle\xi,x\rangle}p_t(x)dx = e^{-t\psi(\xi)}, \quad \text{where } \psi(\xi) = \int_{\mathbb{R}^d} (1 - \cos\langle\xi, x\rangle)\, \nu(dx).$$

Here $\mathbb{E}$ is the integration with respect to $\mathbb{P}$. We note that the Lévy-Khintchine exponent ψ is radial with the radial profile $\psi(\theta) = \psi(\xi)$ for $\xi \in \mathbb{R}^d$, $|\xi| = \theta$.

We estimate the *heat kernel* $p_D(t, x, y)$, $t > 0$, $x, y \in D$, of smooth open sets $D \subset \mathbb{R}^d$ for the operator L. Our estimates have a form of explicit factorization involving the transition density $p_t(x)$ of the Lévy process on the whole of $\mathbb{R}^d$, and the *survival probability* $\mathbb{P}^x(\tau_D > t)$, where $\tau_D = \inf\{t > 0 : X_t \notin D\}$ is the time of the first exit of X_t from D. Here $x \in \mathbb{R}^d$ and $\mathbb{P}^x$ is the law of $x + X$. For instance, if the radial profile of ψ has the so-called *global lower and upper scalings*, and D is a C^2 halfspace-like open set, then

$$p_D(t,x,y) \;\approx\; \mathbb{P}^x(\tau_D > t)\; p(t,x,y)\; \mathbb{P}^y(\tau_D > t), \qquad t>0,\; x,y \in D.$$

Here,

$$p_t(x) \approx \; \psi^{-1}\left(1/t\right)^d \;\wedge\; \frac{t\,\psi(1/|x|)}{|x|^d},$$

$$\mathbb{P}^x(\tau_D > t) \;\approx\; 1 \;\vee\; \left(\psi(1/\delta_D(x))t\right)^{-1/2},$$

and $\delta_D(x) = \mathrm{dist}(x, D^c)$. The scaling conditions are understood as follows.[1] We say that ψ satisfies the weak *lower* scaling condition at infinity (WLSC) if there are numbers $\underline{\alpha} > 0$, $\underline{\theta} \geq 0$ and $\underline{c} \in (0,1]$, such that

$$\psi(\lambda\theta) \geq \underline{c}\lambda^{\underline{\alpha}}\psi(\theta) \quad \text{for} \quad \lambda \geq 1, \quad \theta > \underline{\theta}.$$

We write $\psi \in \mathrm{WLSC}(\underline{\alpha}, \underline{\theta}, \underline{c})$ or $\psi \in \mathrm{WLSC}$. If $\psi \in \mathrm{WLSC}(\underline{\alpha}, 0, \underline{c})$, then we say that ψ satisfies the global WLSC. The weak *upper* scaling condition at infinity (WUSC) means that there are numbers $\overline{\alpha} < 2$, $\overline{\theta} \geq 0$ and $\overline{C} \in [1,\infty)$ such that

$$\psi(\lambda\theta) \leq \overline{C}\lambda^{\overline{\alpha}}\psi(\theta) \quad \text{for} \quad \lambda \geq 1, \quad \theta > \overline{\theta}.$$

In short, $\psi \in \mathrm{WUSC}(\overline{\alpha}, \overline{\theta}, \overline{C})$ or $\psi \in \mathrm{WUSC}$. Global WUSC means $\mathrm{WUSC}(\overline{\alpha}, 0, \overline{C})$. Explicit estimates are also given for bounded smooth sets D, etc.

References

[1] Krzysztof Bogdan, Tomasz Grzywny, and Michał Ryznar, *Density and tails of unimodal convolution semigroups*, J. Funct. Anal. **266** (2014), no. 6, 3543–3571. MR 3165234

[2] ______, *Dirichlet heat kernel for unimodal Lévy processes*, Stochastic Process. Appl. **124** (2014), no. 11, 3612–3650. MR 3249349

[3] ______, *Barriers, exit time and survival probability for unimodal Lévy processes*, Probab. Theory Related Fields **162** (2015), no. 1-2, 155–198. MR 3350043

[1] The notation that follows is a bit heavy, but it quantifies monotonicity properties of $\psi(\lambda)/\lambda^{\alpha}$.

Abed Boulouz
Ibn Zohr University, Morocco

On norm continuity, differentiability and compactness of perturbed semigroups

The object of this talk is to treat regularity, like norm continuity, compactness and differentiability for perturbed semigroups in Banach spaces. In particular, we investigate three large classes of perturbations, Miyadera-Voigt, Desch-Schappacher and Staffans-Weiss perturbations. Our approach is mainly based on feedback theory of Salamon-Weiss systems. Our results are applied to abstract boundary integro-differential equations in Banach spaces.(Joint work with H. Bounit, A. Driouich and S. Hadd)

References

[1] A. Boulouz, H. Bounit, A. Driouich and S. Hadd, *On norm continuity, differentiability and compactness of perturbed semigroups*, submitted.

[2] S. Hadd, R. Manzo and A. Rhandi, *Unbounded perturbations of the Generator domain*, Discrete and Continuous Dynamical Systems, 35 (2015), 703-723.

Henryk Gacki
University of Silesia, Poland
Roksana Brodnicka
University of Silesia, Poland

Asymptotic stability of the evolutionary Boltzmann - type equations

Some problems of the mathematical physics can be written as differential equations for functions with values in the space of measures. The vector space of signed measures doesn't have good analytical properties. For example, this space with the Fortet-Mourier metric is not complete.

There are two methods to overcome this problem. First, we may replace the original equations by the adjoint ones on the space of continuous bounded functions. Secondly, we may restrict our equations to some complete convex subsets of the vector space of measures. This approach seems to be quite natural and it is related to the classical results concerning differential equations on convex subsets of Banach spaces (see [2]). The convex sets method in studying the Boltzmann equation was used in a series of papers (see for example : [1, 3, 5, 6, 7]).

The main purpose of our study is to show some application of the Kantorovich-Rubinstein maximum principle concerning the properties of probability metrics. Then we show that the Kantorovich-Rubinstein maximum principle combined with the LaSalle invariance principle used in the theory of dynamical systems allow us to find new sufficient conditions for the asymptotic stability of solutions of a same version of the nonlinear Boltzmann-type equation. This equation was stimulated by the problem of the stability of solutions of the following version of the Boltzmann equation (see [3])

$$\frac{d\psi}{dt} + \psi = P\,\psi \qquad \text{for} \qquad t \geq 0 \tag{1}$$

with the initial condition

$$\psi\,(0) = \psi_0, \tag{2}$$

where $\psi_0 \in \mathcal{M}_1(\mathbb{R}_+)$ and $\psi : \mathbb{R}_+ \to \mathcal{M}_{sig}(\mathbb{R}_+)$ is an unknown function. Moreover P is the collision operator acting on the space of probability measures.

We will discuss a equation drawn from the kinetic theory of gases which will be a generalized version of (1) (see [4]). We will assume that the collision operator P is a convex combination of N operators $P_1, ..., P_N$, where P_k for $k \geq 2$ describes the simultaneous collision of k particles and P_1 the influence of external forces.

The same equation which was discussed by Lasota [5]. This stability result was based on the technique of Zolotarev metrics. Lasota showed that the stationary solution is exponentially stable in the Zolotarev norm of order 2.

The basic idea of our method is to apply the Kantorovich-Rubinstein maximum principle combined with the LaSalle invariance principle. More precisely we will show that if our equation has a stationary measure $\mu_\star$ such that supp$\mu_\star = \mathbb{R}_+$, then this measure is asymtotically stable with respect to the Kantorovich-Wasserstein metric.

The open problem related with characterization of the stationary measure $\mu_\star$ of the equation (1) will end the talk.

Refer to the positions in the reference list as follows: [1], [2].

References

[1] R. Brodnicka and H. Gacki, *Asymptotic stability of a linear boltzmann-type equation*, Appl. Math. **41** (2014), 323–334.

[2] M. G. Crandall, *Differential equations on convex sets*, J. Math. Soc. Japan **22** (1970), 443–455.

[3] H. Gacki, *Applications of the kantorovich–rubinstein maximum principle in the theory of markov semigroups*, Dissertationes Math. **448** (2007), 1–59.

[4] H. Gacki and Ł. Stettner, *Asymptotic stability of some nonlinear evolutionary equation*, to appear.

[5] A. Lasota, *Asymptotic stability of some nonlinear boltzmann-type equations*, J. Math. Anal. Appl. **268** (2002), 291–309.

[6] A. Lasota and J. Traple, *Asymptotic stability of differential equations on convex sets*, J. Dynam. Differential Equations **15** (2003), 335–355.

[7] ______, *Properties of stationary solutions of a generalized tjon–wu equation*, J. Math. Anal. Appl. **335** (2007), 669 – 682.

Christian Budde
University of Wuppertal, Germany

Extrapolation spaces and Desch–Schappacher perturbations of bi-continuous semigroups

We construct extrapolation spaces for non-densely defined (weak) Hille–Yosida operators. In particular, we discuss extrapolation of bi-continuous semigroups. As an application we present a Desch–Schappacher type perturbation result for this kind of semigroups. This talk is based on [1] and [2], which are joint works with B. Farkas.

References

[1] Christian Budde and Bálint Farkas, *Intermediate and extrapolated spaces for bi-continuous semigroups*, arXiv:1711.05528 [math.FA], 2017.

[2] ———, *Desch–Schappacher perturbations for bi-continuous semigroups*, preprint, 2018.

Yana Butko (Kinderknecht)
University of Saarland, Germany

Chernoff approximation of evolution semigroups and beyond.

We present a method to approximate evolution semigroups with the help of the Chernoff theorem. We discuss different approaches to construct Chernoff approximations for Feller semigroups and Schrödinger groups. In particular, we outline the techniques based on pseudo-differential operators, shifts, rotation. We show how to approximate semigroups obtained from some original (known or already approximated) ones by such procedures as additive and/or multiplicative perturbations of generators, subordination, adding Dirichlet boundary/external conditions ($\sim$ killing of underlying stochastic processes). The described approaches allow to approximate semigroups generated, e.g., by subordinate Feller diffusions on star graphs and Riemannian manifolds. Moreover, the constructed Chernoff approximations for evolution semigroups can be used further to approximate solutions of some time-fractional evolution equations describing anomalous diffusion (solutions of such equations do not posess the semigroup property).

Many Chernoff approximations lead to representations of solutions of (corresponding) evolution equations in the form of limits of n-fold iterated integrals of elementary functions when n tends to infinity. Such representations are called *Feynman formulae*. They can be used for direct computations, modelling of the related dynamics, simulation of stochastic processes. Furthermore, the limits in Feynman formulae sometimes coincide with path integrals with respect to probability measures (such path integrals are usually called *Feynman-Kac formulae*) or with respect to Feynman type pseudomeasures (such integrals are *Feynman path integrals*). Therefore, the constructed Feynman formulae can be used to approximate (or even sometimes to define) the corresponding path integrals; different Feynman formulae for the same semigroup allow to establish connections between different path integrals. Moreover, in some cases, Feynman formulae provide Euler–Maruyama schemes for SDEs; some Chernoff approximations can be understood as a version of the operator splitting method (known in the numerics of PDEs).

References

[1] Ya. A. Butko. Chernoff approximation for semigroups generated by killed Feller processes and Feynman formulae for time-fractional Fokker–Planck–Kolmogorov equations. Preprint. 2017. URL: https://arxiv.org/pdf/1708.02503.pdf.

[2] Ya. A. Butko. Chernoff approximation of subordinate semigroups. *Stoch. Dyn.* 1850021 (2017),19 p., DOI: 10.1142/S0219493718500211.

[3] Ya. A. Butko, M. Grothaus and O.G. Smolyanov. Feynman formulae and phase space Feynman path integrals for tau-quantization of some Lévy-Khintchine type Hamilton functions. *J. Math. Phys.* **57** 023508 (2016), 22 p.

[4] Ya. A. Butko. Description of quantum and classical dynamics via Feynman formulae. *Mathematical Results in Quantum Mechanics: Proceedings of the QMath12 Conference*, p.227-234. World Scientific, 2014. ISBN: 978-981-4618-13-7 (hardcover), ISBN: 978-981-4618-15-1 (ebook).

[5] Ya. A. Butko, R.L. Schilling and O.G. Smolyanov. Lagrangian and Hamiltonian Feynman formulae for some Feller semigroups and their perturbations, *Inf. Dim. Anal. Quant. Probab. Rel. Top.*, **15** N 3 (2012), 26 p.

[6] Ya. A. Butko, M. Grothaus and O.G. Smolyanov. Lagrangian Feynman formulae for second order parabolic equations in bounded and unbounded domains, *Inf. Dim. Anal. Quant. Probab. Rel. Top.* **13** N3 (2010), 377-392.

[7] Ya. A. Butko. Feynman formulas and functional integrals for diffusion with drift in a domain on a manifold, *Math. Notes* **83** N3 (2008), 301–316.

Evgeniy Bychkov
South Ural State University, Russia

Semilinear Sobolev Type Equations of Higher Order

Of concern is the semilinear Sobolev type equation of higher order with Cauchy and Showalter-Sidorov initial conditions. A unique solvability of the problem is proved. We use the ideas and techniques developed by G.A. Sviridyuk for the investigation of the Cauchy problem for a class of the first order semilinear Sobolev type equations and by A.A. Zamyshlyaeva for the investigation of the higher order linear Sobolev type equations. We consider two cases. The first one concerns the case when an operator A at the highest time derivative is continuously invertible. In this case for any initial data from the tangent bundle of the original Banach space there exists a unique solution lying in this space as a trajectory. The second case, when the operator A is not continuously invertible is of great interest for us. Here we use the phase space method. It consists in reducing a singular equation to a regular one which is defined on a subset of the original Banach space consisting of admissible initial values which is understood as a phase space. Under the condition of polynomial boundedness of operator pencil in the case where infinity is a removable singularity of its A-resolvent, a set, which is locally a phase space of the original equation, is constructed. Abstract results are applied to investigation of mathematical model of vibration in the DNA molecule and Bussinesq-Löve mathematical model.

References

[1] E.V. Bychkov A.A. Zamyshlyaeva, *The cauchy problem for the second order semilinear sobolev type equation*, Global and Stochastic Analysis **2** (2015), no. 2, 159–166.

[2] S. Lang, *Introduction to differentiable manifolds*, Springer, NY, 2002.

[3] G. Sviridyuk and T. Sukacheva, *Galerkin approximations of singular nonlinear equations of sobolev type*, Russian Mathematics **33** (1989), no. 10, 56–59.

[4] A.A. Zamyshlyaeva, *The higher-order sobolev-type models*, Bulletin of the South Ural State University, Series: Mathematical Modelling, Programming and Computer Software **7** (2014), no. 2, 5–28.

Waldemar Cieślak
Lublin University of Technology, Poland
Witold Mozgawa
Maria Curie-Sklodowska University, Poland

The Fuss formulas in the Poncelet porism

In this paper we give a proof of Poncelet's closure theorem for ring domains using elementary functions and a certain differential equation which has a solution with suitable geometric properties. We give a necessary and sufficient condition of existence of a constant solution of the equation which explains the phenomenon of the Poncelet porism. In the last section we present a method of determination of the Fuss formulas for an arbitrary natural n. Additionally this method allows us to find the Fuss formulas for closed n-gons with self-intersections.

Filippo Dell'Oro
Polytechnic University of Milan, Italy

Decay properties of dissipative systems of linear thermoelasticity and viscoelasticity

We discuss the stability properties of two dissipative abstract systems, arising from the theory of linear thermoelasticity and viscoelasticity, respectively. In particular, we analyze how the asymptotic behaviour of the associated solution semigroups is influenced by the structural quantities of the problem. These results have been obtained in collaboration with V. Danese and V. Pata [1, 2].

References

[1] V. Danese and F. Dell'Oro, *The lack of exponential stability for a class of second-order systems with memory*, Proc. Roy. Soc. Edinburgh Sect. A **147** (2017), 683–702.

[2] V. Danese, F. Dell'Oro, and V. Pata, *Stability analysis of abstract systems of Timoshenko type*, J. Evol. Equ. **16** (2016), 587–615.

Viktor I. Gerasimenko
National Academy of Sciences of Ukraine, Ukraine

On Semigroups of Operators Describing Processes of Creation and Propagation of Quantum Correlations

The subject of the talk is the analysis of processes of the creation and the propagation of correlations in large particle quantum systems.

We establish properties of cumulants (semi-invariants) of semigroups of operators of finitely many quantum particles that are the generating operators of solution expansions for hierarchies of evolution equations, describing the evolution of quantum correlations, in particular the von Neumann hierarchy for correlation operators [1] and the nonlinear quantum BBGKY (Bogolyubov–Born–Green–Kirkwood–Yvon) hierarchy for marginal correlation operators [3].

Moreover, we consider a mean field scaling behavior of processes of the creation of correlations and the propagation of initial correlations in large particle quantum systems. We establish that such processes are governed by the Vlasov-type quantum kinetic equation with initial correlations [2].

References

[1] V.I. Gerasimenko, *Hierarchies of quantum evolution equations and dynamics of many-particle correlations, in: Statistical mechanics and random walks: Principles, processes and applications*, Nova Science Publ., Inc., N.Y., 2013.

[2] ———, *New approach to derivation of quantum kinetic equations with initial correlations*, Carpathian Math. Publ. **7** (2015), no. 1, 38–48.

[3] ———, *The evolution of correlation operators of large particle quantum systems*, Methods Funct. Anal. Topology **23** (2017), no. 2, 123–132.

Jochen Glück

Convergence of Positive Semigroups and Hyper-Bounded Operators

If $(T_t)_{t\in(0,\infty)}$ is a positive and bounded operator semigroup on L^p and if the essential spectral radius $r_e(T_t)$ of at least one operator T_t is strictly smaller than 1, a classical theorem of Lotz ensures operator norm convergence of T_t as time tends to infinity. This is one reason (among many others) why one is interested in criteria which ensure the property $r_e(T) < 1$ for a positive operator T.

Let (Ω, μ) be a finite measure space and let $p \in (1, \infty)$. A bounded linear operator T on $L^p := L^p(\Omega, \mu)$ is called *hyper-bounded* if $TL^p \subseteq L^q$ for some $q > p$. In 2015 L. Miclo [1] showed that a positive and hyper-bounded operator T on L^2 fulfils $r_e(T) < 1$ in case that T has spectral radius 1 and is self-adjoint (and fulfils a few technical assumptions); this solved a long open conjecture of Høegh-Krohn and Simon.

In this talk we demonstrate that the same theorem remains true under more general assumptions: a positive and hyper-bounded operator T on L^p which is merely power-bounded always fulfils $r_e(T) < 1$. Our methods are very different from Miclo's; we rely on an ultra power technique, combined with the fact that L^p and L^q are not isomorphic for $p \neq q$.

References

[1] Laurent Miclo, *On hyperboundedness and spectrum of Markov operators*, Invent. Math. **200** (2015), no. 1, 311–343.

Gisèle Ruiz Goldstein
University of Memphis, USA

New Results on Instantaneous Blowup in $\mathbb{H}^N$

Consider the heat equation

$$\frac{\partial u}{\partial t} = \Delta u + V(x)u$$

for $x \in \mathbb{R}^N$ with a positive potential $V(x)$. If V is "too singular", then this equation may not have any positive solutions, as was discovered in 1984. We shall discuss the history of the problem as well as later developments, including new results obtained in 2017. The Euclidean space $\mathbb{R}^N$ can be replaced by the Heisenberg group $\mathbb{H}^N$ and other Carnot groups, and the heat equation can be relaced by the Ornstein-Uhlenbeck equation and other related equations. Some nonlinear results will be mentioned. Scaling plays a critical role.

Jerry Goldstein
University of Memphis, USA

The Agmon-Douglis-Nirenberg Problem for Dynamic Boundary Conditions

Of concern are certain reaction-diffusion systems with total mass bounded in the L1 norm. The solution of this problem requires new results from the study of a linear uniformly parabolic heat equation on a bounded domain with dynamic (or Wentzell) boundary conditions incorporating the Laplace-Beltrami operator. We prove that the semigroup governing this problem is analytic in the right half plane in Lp for all p ≥ 1 and for C in the supremum norm. The proof is long and delicate. This is joint work with Gisele Ruiz Goldstein and Michel Pierre.

Federica Gregorio
University of Salerno, Italy

Bi-Laplacians on graphs and networks

We study the differential operator $A = \frac{d^4}{dx^4}$ acting on a connected network $\mathcal{G}$ along with $\mathcal{L}^2$, the square of the discrete Laplacian acting on a connected combinatorial graph G. For both operators we discuss self-adjointness issues, well-posedness of the associated linear parabolic problems

$$\frac{df}{dt} = -\mathcal{L}^2 f, \qquad \frac{\partial u}{\partial t} = -Au,$$

and extrapolation of the generated semigroups to consistent families of semigroups on $L^p(\mathcal{G})$ or $\ell^p(\mathsf{V})$, respectively, for $1 \leq p \leq \infty$. Our most surprising finding is that, upon allowing the system enough time to reach diffusive regime, the parabolic equations driven by $-A$ may display Markovian features: analogous results seem to be unknown even in the classical case of domains. This kind of analysis is based on a detailed study of bi-harmonic functions complemented by simple combinatorial arguments; we emphasize the role of boundary conditions in the vertices of the network, and in turn of the network's connectivity, in determining such Markovian properties. We elaborate on analogous issues for the discrete bi-Laplacian; a characterization of complete graphs in terms of the Markovian property of the semigroup generated by $-\mathcal{L}^2$ is also presented.

Joint work with Delio Mugnolo.

Adam Gregosiewicz
Lublin University of Technology, Poland

Asymptotics of the Lebowitz–Rubinow–Rotenberg model of cell populations development

We consider a mathematical model of cell populations dynamics proposed by J. Lebowitz and S. Rubinow, and analysed by M. Rotenberg. In the model each cell is characterized by her maturity and speed of maturation. The growth of cell populations is described by a partial differential equation with a boundary condition. More precisely growth of the cells' population density is governed by the partial differential equation

$$\frac{\partial f}{\partial t} = -v\frac{\partial f}{\partial x},$$

where $f = f(x, v, t)$ with $t \geq 0$ is the cells' density at (x, v) at time t. In this model a cell starts maturing at $x = 0$ and divides reaching $x = 1$, and the boundary condition

$$vf(0, v, t) = p\int_V wk(w, v)f(1, w, t)dw$$

describes the reproduction rule. Here k satisfies

$$\int_V k(w, v)dv = 1$$

for any $w \in V$, and $V \ni v \mapsto k(w, v)$ is the probability density of the daughter's maturation velocity conditional on w being the velocity of the mother. Furthermore, it is assumed that $p \geq 0$ is the average number of viable daughters per mitosis.

By applying the Lord Kelvin method of images we give a new proof that the model is well posed. A semi-explicit formula for the semigroup related to the model obtained by the method of images allows also two types of new results. First of all, we give growth order estimates for the semigroup, applicable also in the case of decaying populations. Secondly, we study asymptotic behavior of the semigroup in the case of approximately constant population size. More specifically, we formulate conditions for the asymptotic stability of the semigroup in the case in which the average number of viable daughters per mitosis equals one.

Logan Hart
Louisiana State University, USA

Superstability of Semigroups

We discuss semigroups that decay faster than any exponential exp(-ωt). In particular, we answer a question of A. V. Balakrishnan from 2005 by providing elementary physical examples of superstability which are not of the type of extinction-in-finite-time (i.e., not nilpotent). These examples are less artificial (in the context of semigroup theory) than those given by G. Lumer in 2001/2002 or by F. Udwadia in 2005.

Amru Hussein
Technical University of Darmstadt, Germany

Non-self-adjoint graphs

On finite metric graphs Laplace operators subject to general non-self-adjoint boundary conditions imposed at graph vertices are considered. A regularity criterion is proposed and spectral properties of such regular operators are investigated, in particular similarity transforms to self-adjoint operators. Concrete examples are discussed exhibiting that non-self-adjoint boundary conditions can yield to unexpected spectral features.

The talk is based on joint work [1] with David Krejčiřík (Czech Technical University in Prague) and Petr Siegl (Queen's University Belfast).

References

[1] A. Hussein, D. Krejčiřík, and P. Siegl, *Non-self-adjoint graphs*, Transactions of the American Mathematical Society **4** (2015), no. 367, 2921 – 2957.

Tomasz Jakubowski
Wroclaw University of Science and Technology, Poland

Critical negative Schrödinger perturbations of fractional Laplacian

Let $p(t,x,y)$ be the fundamental solution of the equation

$$\partial_t u(t,x) = \Delta^{\alpha/2} u(t,x).$$

I will consider the integral equation

$$\tilde{p}(t,x,y) = p(t,x,y) + \int_0^t \int_{\mathbb{R}^d} p(t-s,x,z) q(z) \tilde{p}(s,z,y) dz ds,$$

where $q(z) = \frac{\kappa}{|z|^\alpha}$ and κ is some constant. The function $\tilde{p}$ solving this equation will be called the Schrödinger perturbations of the function p by q. The case $\kappa > 0$ where recently studied in [1]. First, I will briefly present the main results of this paper. Next, I will focus on the case of negative κ and present the estimates of the function $\tilde{p}$ for all $\kappa \in (-\infty, 0)$.

References

[1] K. Bogdan, T. Grzywny, T. Jakubowski, and D. Pilarczyk, *Fractional Laplacian with Hardy potential*, preprint (2017).

L.O. Joel
University of Kwazulu-Natal, South Africa
J. Banasiak
University of Pretoria, South Africa
Technical University of Łódź, Poland
S. Shindin
University of Kwazulu-Natal, South Africa

The Discrete Unbounded Coagulation-Fragmentation Equation with Growth, Decay and Sedimentation

We study the discrete coagulation–fragmentation models with growth, decay and sedimentation. We demonstrate the existence and uniqueness of classical global solutions to these models using the theory of semigroups of operators. Theoretical conclusions are supported by numerical simulations. [1], [2].

References

[1] J. Banasiak, L.O. Joel, and S. Shindin, *Analysis and simulations of the discrete fragmentation equation with decay*, Mathematical Methods in the Applied Sciences (2017).

[2] ———, *Long term dynamics of the discrete growth-decay-fragmentation equation*, eprint arXiv:1801.06486, 01/2018, 2018.

Agnieszka Kałamajska
Polish Academy of Sciences, Poland
University of Warsaw, Poland

Dirichlet's problem for critical Hamilton-Jacobi fractional equation

Using an extended approach of Dan Henry, we study solvability of the Dirichlet problem on a bounded smooth domain for the Hamilton-Jacobi equation with critical nonlinearity posed in Sobolev spaces:

$$\begin{cases} u_t + (-\Delta)^{1/2}u + H(u, \nabla u) = 0, t > 0, x \in \Omega, \\ u(t, x) = 0, t > 0, x \in \partial\Omega, \\ u(0, x) = u_0, x \in \Omega. \end{cases}$$

We will also discuss the additional regularity and uniqueness of the limiting weak solution. The talk will be based on joint work with Tomasz Dlotko.

Michael Kaplin
University of Ljubljana, Slovenija

Relatively uniformly continuous semigroups on vector lattices

We shall introduce and study the notion of relatively uniformly continuous semigroups on vector lattices. We will present some basic results for such semigroups. In particular, we will see an analogue of the Hille-Yosida Theorem for relatively uniformly continuous positive semigroups on vector lattices which satisfy a Banach-Steinhaus property and are relatively uniformly complete.

Bogdan Kaźmierczak
Polish Academy of Sciences, Poland
Paramita Chatterjee
Polish Academy of Sciences, Poland

Existence of solutions in a model of bone pattern formation

We consider a relatively new model of bone structure formation during morphogenesis based upon a specific interaction between galectin proteins with their membrane receptors proposed in [1]. This model is governed, in general, by a system of parabolic and parabolic-hyperbolic equations. Upon some simplifications, we use a modification of the Rothe method to prove the existence of solutions. We also justify partially the approximation of the system by a set of parabolic equations.

References

[1] T. Glimm, R. Bhat, and S.A. Newman, *Modeling the morphodynamic galectin patterning network of the developing avian limb skeleton*, J. Theor. Biol. **346** (2014), 86–108.

Alevtina Keller
South Ural State University, Russia

Algorithms for the Numerical Solution of Optimal Control Problems for Models of the Leontief Type

The report provides an overview of results of a numerical research of the class of problems that is considered by the example of optimal control, hard control, start control and hard starting control for the Leontief type systems. Such systems arise in modelling of different processes and objects, for example measuring transducers, economic systems of an enterprise, and dynamics of a cell cycle. Leontief type system is a finite-dimensional analogue of the Sobolev type equation therefore our research is based on the methods of the theory of degenerate groups of operators. The report presents an algorithm for finding of approximate solutions to a variety of optimal control problems for Leontief type systems with the Showalter–Sidorov initial condition which is more convenient in numerical research. The proof of the convergence of the approximate solutions to the precise one is an important result. The issues of improving of the efficiency of numerical algorithms and their modifications in the numerical study of applications are discussed. Special attention is given to the numerical algorithms for solving of the optimal measurement problems which are the problems of restoration of signals dynamically distorted both by inertia of the measuring device, and resonances in its circuits. The results of computational experiments are presented.

James Kennedy
University of Lisbon, Portugal

Hot spots of quantum graphs

Let $(e^{t\Delta_\Omega^N})_{t\geq 0}$ denote the semigroup associated with the Neumann Laplacian on a bounded Euclidean domain $\Omega \subset \mathbb{R}^d$. The Hot Spots Conjecture of J. Rauch asserts that for a "generic" initial condition u_0, if $x_t \in \overline{\Omega}$ is any point at which $u(t,x) := e^{t\Delta_\Omega^N} u_0(x)$ reaches is maximum (or minimum), then $x_t \to \partial\Omega$ as $t \to \infty$. In words, the hottest and coldest points of the body Ω should generically move towards its boundary for large times, if the insulation is perfect. In its most common formulation, this reduces to proving that maximum and minimum of the eigenfunction(s) associated with the smallest positive eigenvalue μ_2 of $-\Delta_\Omega^N$ are located on the boundary. This conjecture is not true in full generality [2] but is currently open, for example, for convex domains [1, 3].

In this talk we will examine the corresponding question on metric graphs: if μ_2 denotes the smallest positive eigenvalue of the Laplacian with standard (continuity and Kirchhoff) vertex conditions, we consider the possible distribution of maxima and minima of eigenfunctions associated with μ_2. Among other things, we give examples to show that the usual notion of "boundary" of a metric graph, namely the set of vertices of degree one, has limited relevance for determining the "hottest" and "coldest" parts of a graph.

This is based on ongoing joint work with Jonathan Rohleder (University of Stockholm), see also [4].

References

[1] R. Bañuelos and K. Burdzy, *On the hot spots conjecture of J. Rauch*, J. Funct. Anal. **164** (1999), 1–33.

[2] K. Burdzy and W. Werner, *A counterexample to the "Hot Spots" Conjecture*, Ann. Math. **149** (1999), no. 1, 309–317.

[3] C. Judge and S. Mondal, *Euclidean triangles have no hot spots*, preprint (2018), arXiv:1802.01800.

[4] J. Kennedy and J. Rohleder, *On the hot spots of quantum trees*, preprint (2018), arXiv:1808.02274.

Lyndsay Kerr
University of Strathclyde, United Kingdom

Joint work with **Wilson Lamb** and **Matthias Langer**, University of Strathclyde, United Kingdom

Discrete Coagulation-Fragmentation Systems

In many situations in nature and industrial processes, clusters of particles can combine into larger clusters or fragment into smaller clusters. The evolution of these clusters can be described by differential equations known as coagulation-fragmentation equations. In the discrete size case it is assumed that the mass of each cluster is a natural number and a cluster of mass n consists of n identical units. The main part of the talk will concentrate on the case of pure discrete fragmentation.

The fragmentation system will be written as an abstract Cauchy problem (ACP) in a weighted ℓ^1 space, with weight $(w_n)_{n=1}^{\infty}$. Previous investigations, [1], [2], have considered the space where the weight is of the form $w_n = n^p$ for some $p \geq 1$. When $p = 1$ the norm reflects the total mass in the system of clusters and so this is the most physically relevant space to work in. However, we have also found it useful to examine the system in weighted ℓ^1 spaces with more general weights. In particular, for any fragmentation rates, we can always find a weighted ℓ^1 space such that there exists an analytic semigroup related to the fragmentation ACP. This result is obtained using perturbation results, which cannot be applied when the weight is of the form $w_n = n$. Using this analytic semigroup, and a Sobolev tower construction, we examine the existence and uniqueness of solutions to the fragmentation system.

The full coagulation-fragmentation system, where the coagulation coefficients may be time-dependent, may also be briefly examined.

References

[1] J. Banasiak, *Global classical solutions of coagulation-fragmentation equations with unbounded coagulation rates*, Global classical solutions of coagulation-fragmentation equations with unbounded coagulation rates **13** (2012), 91–105.

[2] A.C. MxBride, A.L. Smith, and W. Lamb, *Strongly differentiable solutions of the discrete coagulation-fragmentation equation*, Phys. D **239** (2010), no. 15, 1436–1445.

Damir Kinzebulatov
Université Laval, Canada

A new approach to the L^p theory of $-\Delta + b\cdot\nabla$ and its application to Feller processes with general drifts

I will discuss joint results with Yu. A. Semenov (Toronto). In $\mathbb{R}^d$, $d \geqslant 3$, consider the following classes of vector fields:

(1) We say that a $b : \mathbb{R}^d \to \mathbb{C}^d$ belongs to the Kato class $\mathbf{K}^{d+1}_\delta$, and write $b \in \mathbf{K}^{d+1}_\delta$, if $|b| \in L^1_{\rm loc}$ and there exists $\lambda = \lambda_\delta > 0$ such that

$$\|b(\lambda-\Delta)^{-\frac{1}{2}}\|_{1\to 1} \leqslant \delta.$$

(2) We say that a $b : \mathbb{R}^d \to \mathbb{C}^d$ belongs to $\mathbf{F}_\delta$, the class of form-bounded vector fields, and write $b \in \mathbf{F}_\delta$, if $|b| \in L^2_{\rm loc}$ and there exists $\lambda = \lambda_\delta > 0$ such that

$$\|b(\lambda-\Delta)^{-\frac{1}{2}}\|_{2\to 2} \leqslant \sqrt{\delta}.$$

(3) We say that a $b : \mathbb{R}^d \to \mathbb{C}^d$ belongs to $\mathbf{F}^{\frac{1}{2}}_\delta$, the class of *weakly* form-bounded vector fields, and write $b \in \mathbf{F}^{1/2}_\delta$, if $|b| \in L^1_{\rm loc}$ and there exists $\lambda = \lambda_\delta > 0$ such that

$$\||b|^{\frac{1}{2}}(\lambda-\Delta)^{-\frac{1}{4}}\|_{2\to 2} \leqslant \sqrt{\delta}.$$

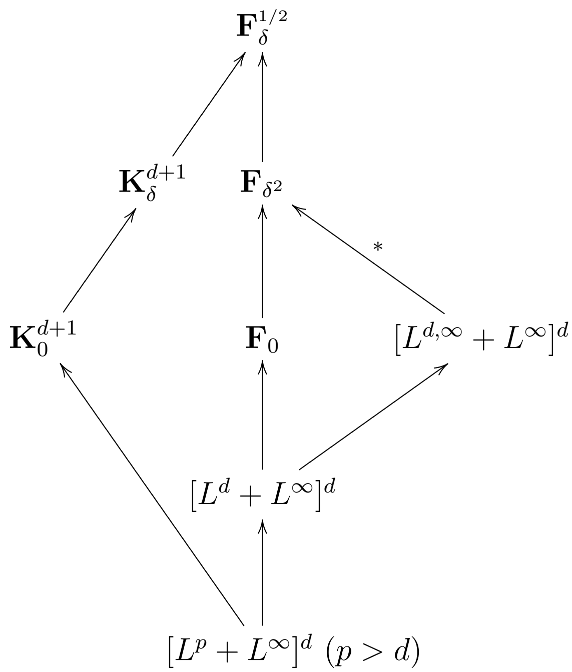
The classes $\mathbf{F}_\delta$, $\mathbf{K}^{d+1}_\delta$ cover singularities of b of critical order, at isolated points or along hypersurfaces, respectively. Both classes have been thoroughly studied in the literature: after 1996, the Kato class $\mathbf{K}^{d+1}_\delta$, with $\delta > 0$ sufficiently small (yet allowed to be non-zero), has been recognized as 'the right' class for the Gaussian upper and lower bounds on the fundamental solution of $\partial_t - \Delta + b\cdot\nabla$ which, in turn, allow to construct an associated Feller semigroup (in C_b). The class $\mathbf{F}_\delta$, $\delta < 4$, is responsible for dissipativity of $\Delta - b\cdot\nabla$ in L^p, $p \geqslant \frac{2}{2-\sqrt{\delta}}$, needed to run an iterative procedure taking $p \to \infty$ (assuming additionally $\delta < \min\{4/(d-2)^2, 1\}$), which produces an associated Feller semigroup in C_∞; see [1] for details. We emphasize that, in general, the Gaussian bounds are not valid if $|b| \in L^d$ ($\subsetneq \mathbf{F}_0 := \bigcap_{\delta>0}\mathbf{F}_\delta$), while $b \in \mathbf{K}^{d+1}_0$ ($:= \bigcap_{\delta>0}\mathbf{K}^{d+1}_\delta$), in general, destroys L^p-dissipativity.

The class $\mathbf{F}^{1/2}_\delta$ contains both classes $\mathbf{F}_\delta$ and $\mathbf{K}^{d+1}_\delta$:

$$\mathbf{K}^{d+1}_\delta \subsetneq \mathbf{F}^{1/2}_\delta, \qquad \mathbf{F}_{\delta_1} \subsetneq \mathbf{F}^{1/2}_\delta \quad \text{for } \delta = \sqrt{\delta_1},$$

$$\left(b \in \mathbf{F}_{\delta_1} \text{ and } \mathsf{f} \in \mathbf{K}^{d+1}_{\delta_2}\right) \Longrightarrow \left(b + \mathsf{f} \in \mathbf{F}^{1/2}_\delta,\ \sqrt{\delta} = \sqrt[4]{\delta_1} + \sqrt{\delta_2}\right)$$

To deal with such general class of vector fields we will use ideas of E. Hille and J. Lions (alternatively, ideas of E. Hille and H.F. Trotter) to construct the generator $-\Lambda \equiv -\Lambda(b)$ (an operator realization of $\Delta - b\cdot\nabla$) of a quasi bounded holomorphic semigroup in L^2. This operator has some remarkable properties. Namely,



General classes of vector fields $b : \mathbb{R}^d \to \mathbb{R}^d$ studied in the literature in connection with operator $-\Delta + b \cdot \nabla$. Here $\to$ stands for strict inclusion, and $\overset{*}{\to}$ reads "if $b = b_1 + b_2 \in [L^{d,\infty} + L^\infty]^d$, then $b \in \mathbf{F}_{\delta^2}$ with $\delta > 0$ determined by the value of the $L^{d,\infty}$-norm of $|b_1|$,

$\rho(-\Lambda) \supset \mathcal{O} := \{\zeta \mid \operatorname{Re}\zeta > \lambda\}$, and, for every $\zeta \in \mathcal{O}$,

$$\begin{aligned}
(\zeta + \Lambda)^{-1} =& J_\zeta^3(1 + H_\zeta^* S_\zeta))^{-1} J_\zeta \\
=& J_\zeta^4 - J_\zeta^3 H_\zeta^* (1 + S_\zeta H_\zeta^*)^{-1} S_\zeta J_\zeta; \\
\|H_\zeta^* S_\zeta\| \leq& \delta, \quad \|(\zeta + \Lambda)^{-1}\|_{2\to 2} \leq |\zeta|^{-1}(1-\delta)^{-1}; \\
\|e^{-t\Lambda_r}\|_{r \to q} \leq& c\, e^{t\lambda} t^{-\frac{d}{2}(\frac{1}{r} - \frac{1}{q})}, \quad 2 \leq r < q \leq \infty;
\end{aligned}$$

where $J_\zeta := (\zeta - \Delta)^{-\frac{1}{4}},\ H_\zeta := |b|^{\frac{1}{2}} J_{\bar\zeta},\ S := b^{\frac{1}{2}} \cdot \nabla J_\zeta^3,\ b^{\frac{1}{2}} := |b|^{-\frac{1}{2}} b.$

As in the case $b \in \mathbf{F}_\delta$, it is reasonable to expect that there exists a strong dependence between the value of δ (effectively playing the role of a "coupling constant" for $b \cdot \nabla$) and smoothness of the solutions to the equation $(\zeta + \Lambda_r)u = f,\ \zeta \in \rho(-\Lambda_r),\ f \in L^r$ ($-\Lambda_r \equiv -\Lambda_r(b)$ is an operator realization of $\Delta - b \cdot \nabla$). Such a dependence does exist. Set

$$m_d := \pi^{\frac{1}{2}}(2e)^{-\frac{1}{2}} d^{\frac{d}{2}} (d-1)^{-\frac{d-1}{2}}, \quad \kappa_d := \frac{d}{d-1}, \quad r_\mp := \frac{2}{1 \pm \sqrt{1 - m_d \delta}}.$$

It will be established that if $b \in \mathbf{F}_\delta^{1/2}$ and $m_d\delta < 1$, then $(e^{-t\Lambda_r(b)},\ r \in [2,\infty[)$ extends by continuity to a quasi bounded C_0 semigroup in L^r for all $r \in]r_-, \infty[$. For every $r \in I_s :=]r_-, r_+[$, the semigroup is holomorphic, the resolvent set $\rho(-\Lambda_r(b))$

contains the half-plane $\mathcal{O} := \{\zeta \in \mathbb{C} \mid \operatorname{Re}\zeta > \kappa_d \lambda_\delta\}$, and the resolvent admits the representation

$$(\zeta + \Lambda_r(b))^{-1} = (\zeta - \Delta)^{-1} - Q_r(1+T_r)^{-1}G_r, \quad \zeta \in \mathcal{O}, \tag{$\star$}$$

where Q_r, G_r, T_r are bounded linear operators on L^r; $D(\Lambda_r(b)) \subset W^{1+\frac{1}{q},r}$ $(q > r)$.

In particular, for $m_d\delta < 4\frac{d-2}{(d-1)^2}$, there exists $r \in I_s$, $r > d-1$, such that $(\zeta + \Lambda_r(b))^{-1}L^r \subset C^{0,\gamma}, \gamma < 1 - \frac{d-1}{r}$.

The results above yield the following: Let $b \in \mathbf{F}_\delta^{1/2}$ for some δ such that $m_d\delta < 4\frac{d-2}{(d-1)^2}$. Let $\{b_n\}$ be any sequence of bounded smooth vector fields, such that $b_n \to b$ strongly in L^1_{loc}, and, for a given $\varepsilon > 0$ and some $\delta_1 \in]\delta, \delta+\varepsilon]$, $\{b_n\} \subset \mathbf{F}_{\delta_1}^{1/2}$. Then

$$s\text{-}C_\infty\text{-}\lim_{n\uparrow\infty} e^{-t\Lambda_{C_\infty}(b_n)} \tag{$\star\star$}$$

exists uniformly in $t \in [0,1]$, and hence determines a Feller semigroup $e^{-t\Lambda_{C_\infty}(b)}$.

The results $(\star)$, $(\star\star)$ can be obtained via direct investigation in L^r of the operator-valued function $\Theta_r(\zeta, b)$ defined by the right hand side of $(\star\star)$ without appealing to L^2 theory (but again appealing to the ideas of E. Hille and H. F. Trotter). See [1] for details.

References

[1] Kinzebulatov D. and Semenov Yu. A. *On the theory of the kolmogorov operator in the spaces l^p and c_∞. i*, Preprint, arXiv:1709.08598 (2017).

Jan Kisyński

Petrovsky condition for forward evolution and semigroups

About ten years ago suddenly I get infected by interest in theory of distributions, especially in rapidly decreasing distributions on $\mathbb{R}^n$ whose set is denoted by $O'_C(\mathbb{R}^n)$. During these ten years I proved the following:

$$O'_C(\mathbb{R}^n)*S(\mathbb{R}^n) \subset L(S(\mathbb{R}^n), S(\mathbb{R}^n)),$$

so that $O'_C(\mathbb{R}^n)$ can be equipped with topology op induced from $L(S(\mathbb{R}^n), S(\mathbb{R}^n))_b$. The locally convex space $(O'_C(\mathbb{R}^n), op)$ is complete. The Fourier transformation yields an isomorphism of locally convex spaces $(O'_C(\mathbb{R}^n), op)$ and $O_M(\mathbb{R}^n)$ such that $F(O'_C(\mathbb{R}^n)*S'(\mathbb{R}^n)) = F(O'_C(\mathbb{R}^n)) \bullet F(S'(\mathbb{R}^n))$. Thanks to the fact that $O_M(\mathbb{R}^n)$ is algebra of multipliers of $S(\mathbb{R}^n)$ the last isomorphism implies at once that for $(O'_C(\mathbb{R}^n), op)$ the fundamental Theorem XV from Chapter VII of the book of L.Schwartz is true. The topology β in $O'_C(\mathbb{R}^n)$, invented originally by L.Schwartz, is strictly finer than the topology op, so that $(O'_C(\mathbb{R}^n), \beta)$ is not isomorphic with $O_M(\mathbb{R}^n)$. The aforementioned relation between the Petrovsky condition for forward evolution and one-parameter convolution semigroups (in $O'_C(\mathbb{R}^n)$ and in $S'(\mathbb{R}^n)$) has secondary importance, but agrees with scope of the conference.

Paweł Klimasara
University of Silesia in Katowice, Poland
Marta Tyran-Kamińska
Polish Academy of Sciences, Poland

A model for random fire induced tree-grass coexistence in savannas

Tree-grass coexistence in savanna ecosystems depends strongly on environmental disturbances out of which crucial is fire. Most modeling attempts in the literature lack stochastic approach to fire occurrences which is essential to reflect their unpredictability. Existing models that actually include stochasticity of fire are usually analyzed only numerically. We introduce a new minimalistic model of tree-grass coexistence where fires occur according to a stochastic process. We use the tools of the stochastic semigroup theory to provide a more careful mathematical analysis of the model. Essentially we show that there exists a unique stationary distribution of tree and grass biomasses.

Victoria Knopova
Technical University of Dresden, Germany

Long-time behaviour of some Markov processes

Let

$$Lf(x) = \int_{\mathbb{R}^d} \big(f(x+u) - f(x) - \nabla f(x) \cdot u 1_{|u|\le 1} \big) \, \nu(x, du), \tag{1}$$

where $\nu(x, du)$ is a Lévy type kernel. Suppose that $(L, C_\infty^2(\mathbb{R}^d))$ extends to a generator of a Feller semigroup $(P_t)_{t\ge 0}$. Denote by X the respective Markov process.

We investigate the sufficient conditions of the recurrence and transience of X. For this we employ the Forster-Lyapunov criterion.

This work is partly motivated by the recent paper of N. Sandrić [1].

References

[1] N. Sandrić, *Long-time behavior of stable-like processes*, Stoch. Proc. Appl. (2013).

Andrzej Komisarski
University of Łódź, Poland

On a model of the ideal heat exchanger and its relation to the telegrapher's equations

We examine a model of the ideal heat exchanger. In the ideal heat exchanger we have two one-dimensional objects (bars or columns of liquid in pipes) of lengths a and b, respectively. The linear heat capacities of the objects are p and q, respectively. The heat is not transferred along the objects. It is only transferred between the objects and the linear thermal conductivity is α (the case $\alpha \to \infty$ is especially interesting). The objects are moving and one passes along another with the constant velocity 1. Let $\widetilde{f} : [0,a] \to \mathbb{R}$ and $\widetilde{g} : [0,b] \to \mathbb{R}$ be continuous functions representing the initial temperature distribution along the objects. We examine the time evolution of the temperature distribution. Our model is described by the following system of equations:

$$\frac{\partial f_\alpha(x,t)}{\partial t} = \begin{cases} \frac{\alpha}{p} \cdot (g_\alpha(t-x,t) - f_\alpha(x,t)) & \text{if } x \in [0,a] \text{ and } t-x \in (0,b) \\ 0 & \text{if } x \in [0,a] \text{ and } t-x \notin [0,b] \end{cases}$$

$$\frac{\partial g_\alpha(x,t)}{\partial t} = \begin{cases} \frac{\alpha}{q} \cdot (f_\alpha(t-x,t) - g_\alpha(x,t)) & \text{if } x \in [0,b] \text{ and } t-x \in (0,a) \\ 0 & \text{if } x \in [0,b] \text{ and } t-x \notin [0,a] \end{cases}$$

where $f_\alpha : [0,a] \times \mathbb{R}_+ \to \mathbb{R}$ and $g_\alpha : [0,b] \times \mathbb{R}_+ \to \mathbb{R}$ and the initial conditions are $f_\alpha(x,0) = \widetilde{f}(x)$ and $g_\alpha(x,0) = \widetilde{g}(x)$. The above system is not autonomous and it does not determine any semigroup. However, it is closely related to the telegrapher's equations (which can be seen after changing the variables), but its behaviour is different. For example, despite the continuity of $\widetilde{f}$ and $\widetilde{g}$, the limits of f_α and g_α for $\alpha \to \infty$ do not need to be continuous.

Jurij Kozicki
Maria Curie-Skłodowska University, Poland

Infinite populations of interacting entities as complex systems: multi-scale Markov dynamics

In many applications, one deals with systems of entities characterized by a trait x belonging to a topological space X. It is believed that a given entity with trait x interacts mostly (even entirely) with those entities whose traits belong to a neighborhood of x. Such interactions form the local structure of the system. The main aim of the talk is to explain how the local structure determines the global behavior of the whole infinite system. In particular, this applies to the Markov evolution of the system's states defined as probability measures on the corresponding configuration spaces. In view of the mentioned complexity, this evolution is considered in different scales that opens the possibility to get a deeper insight into its properties. This approach will be demonstrated in a number of models.

Marjeta Kramar Fijavž
University of Ljubljana, Slovenia

On perturbing the domain of certain generators

Basing on the approach of Greiner (1987), Salomon (1987), Weiss (1994), and Staffans (2005), Adler-Bombieri-Engel (2014) and Hadd-Manzo-Rhandi (2015) obtained very general abstract results on the well-posedness of perturbed Cauchy problems. Using these results we will study first and second order differential operators with variable coefficients and general boundary conditions acting on spaces of L_p-functions defined on a finite union of intervals. We will present conditions on the coefficients and the domain which assure the generation of a C_0-semigroup and hence well-posedness of the corresponding Cauchy problem.

The talk is based on joint work with Klaus-Jochen Engel (L'Aquila).

Henrik Kreidler
Tübingen University, Germany

Weighted Koopman semigroups and their applications

Cocycles over flows arise naturally in many situations, e.g., in the context of evolution families or in smooth ergodic theory. Such cocycles induce strongly continuous weighted Koopman semigroups. In this talk we give abstract characterizations of such semigroups and discuss their properties.

Josef Kreulich

On Compactifications of bounded C_0-semigroups

In this study, we refine the compactification presented by Witz [1] for general semigroups to the case of bounded C_0-semigroups, involving adjoint theory for this class of operators. This approach considerably reduces the operator space where the compactification is constructed. Additionally, this approach leads to a decomposition of $X^{\odot}$ and to an extension of ergodic results to dual semigroups.

Keywords: recurrent vectors, flight vectors, almost automorphic vectors

References

[1] K. Witz, *Applications of a compactification for bounded operator semigroups*, Illinois J. Math. **8** (1964), 685–696.

Wojciech Kryszewski
Technical University of Łódź, Poland
Nicolaus Copernicus University in Toruń, Poland

Bifurcation at infinity for elliptic problems on $\mathbb{R}^N$

In the talk the asymptotic bifurcation of solutions to a parameterized stationary semilinear Schrödinger equation involving a potential of the Kato-Rellich type will be discussed. It will be shown that the bifurcation from infinity occurs if the parameter is an eigenvalue of the hamiltonian lying below the asymptotic bottom of the bounded part of the potential. Thus the bifurcating solution are related to bound states of the corresponding Schrödinger equation. The argument relies on the careful study of the properties of the underlying operator and on the use of the (generalized) Conley index due to Rybakowski and resonance assumptions of the Landesman-Lazer or sign-condition type. This is a research joint with A. Ćwiszewski.

Markus Kunze
University of Konstanz, Germany

Diffusion with nonlocal Dirichlet boundary conditions on unbounded domains.

In [1], we have considered second order strictly elliptic operators of the form

$$\mathcal{A}u = \sum_{i,j=1}^{d} a_{ij}\partial_i\partial_j u + \sum_{j=1}^{d} b_j\partial_j u$$

on a bounded, Dirichlet regular set $\Omega \subset \mathbb{R}^d$. These operators were complemented by nonlocal Dirichelet boundary conditions:

$$u(z) = \int_\Omega u(x)\mu(z, dx), \quad z \in \partial\Omega.$$

These boundary conditions have a clear probabilistic interpretation. When a particle, diffusing according to the operator $\mathcal{A}$, hits the boundary in a point z, it jumps instantly back to the interior. The position it jumps to is chosen according to the distribution $\mu(z,\cdot)$. In [1] we were able to prove that a realization of this operator generates a semigroup on the space $C(\overline{\Omega})$. It should be noted that this semigroup is an analytic semigroup, but*not* strongly continuous.

In this talk we will present an extension of this result where we allow Ω to be an unbounded set and the coefficients of the operator $\mathcal{A}$ to be unbounded near infinity. It should be noted that the semigroup generated by such an operator can no longer be expected to be analytic, so that the rich theory of analytic semigroups is no longer at our disposal to compensate for the lack of strong continuity. Instead, we present an approach based on approximation, where we approximate the unbounded domain Ω with bounded domains Ω_n. We will also discuss additional properties of the semigroup such as the strong Feller property and asymptotic behavior.

References

[1] Wolfgang Arendt, Stefan Kunkel, and Markus Kunze, *Diffusion with nonlocal boundary conditions*, J. Funct. Anal. **270** (2016), no. 7, 2483–2507. MR 3464048

Thomas G. Kurtz
University of Wisconsin - Madison, USA

Generators, martingale problems, and stochastic equations

A natural way of specifying a Markov process is by defining its generator. Classically, one then shows that the generator, or some natural extension, is the generator of a positive, contraction semigroup which determines the transition function of the Markov process which must satisfy the Kolmogorov *forward* and *backward* equations.

Standard semigroup identities and the relationship between the process and the semigroup also imply that the process has certain martingale properties which are the basis for the classical identity known as Dynkin's identity. In work on diffusions, Stroock and Varadhan, exploiting these properties, formulated a *martingale problem* as an approach to uniquely determining the process corresponding to the generator.

For many processes, in particular diffusions, the process can also be determined as a solution of a stochastic equation. Very generally, the forward equation, the martingale problem, and, if one exists, the corresponding stochastic equation, are equivalent in the sense that a solution of one corresponds to solutions of the others. In particular, uniqueness of one implies uniqueness of the other.

The formulation of the three problems becomes more complicated in the case of constrained Markov processes (for example, reflecting diffusions). Associating a constrained martingale problem with a certain controlled martingale problem in a sense reduces the problem of equivalence of the three approaches to specifying the process to the unconstrained case. The forward equations, martingale problems, and (in examples) stochastic equations will be formulated and proof of their equivalence outlined.

Stanisław Kwapień
Polish Academy of Sciences, Poland

Continuity and boundness of stochastic convolutions

We deal with the stochastic convolutions of two types. The first type are the convolutions of the form

$$\int_0^t P(t-s)dX(s), \;\; t \geq 0$$

were $P(t), t \geq 0$ is a continous semigroup of operators in a Hilbert space H and $X(t), t \geq 0$ is a square integrable martingale with values in H.
The second type are convolutions of the form

$$\int_0^t Z(t-s)dX(s), t \geq 0$$

where $Z(t), t \geq 0$ is a real continuous process on R^+ with $Z(0) = 0$ and X is a $X(t), t \geq 0$ is a real square integrable martingale independent of Z. In particular we consider the case in which Z is a deterministic function.

In the both cases we are interested when the convolutions are continuous (or locally bounded) processes. We present some known results as well much more of open questions.

Wilson Lamb
University of Strathclyde, United Kingdom

Discrete Fragmentation Equations

The process of fragmentation arises in many physical situations, including depolymerisation, droplet break-up and rock fracture. In some cases, it is possible to model processes of this type using a discrete size variable. For example, if the fragmenting objects, such as polymers, can be interpreted as clusters composed of identical fundamental units (e.g. monomers), then clusters of size n (n-mers) are those which consist of n fundamental units. Under suitable assumptions, the evolution of the number concentration of clusters of all positive integer sizes can be described by an infinite linear system of ordinary differential equations. In the first part of this talk, it will be shown that the associated initial-value problem for this infinite-dimensional system can be expressed as an abstract Cauchy problem, posed in a physically relevant Banach space. Routine application of a result on stochastic semigroups by Thieme and Voigt then leads to the existence and uniqueness of classical solutions.

Although it is usual to assume that the fragmentation process conserves the total mass of all clusters, there are examples, such as bond annihilation, in which mass loss can arise in a natural manner. The second part of the talk is based on joint work [1] with Louise Smith, Matthias Langer and Adam McBride (University of Strathclyde), and focusses on a simple model of random bond annihilation for which it is possible to establish that the associated fragmentation semigroup is analytic. This model will also be used to highlight the problem of non-uniqueness of pointwise solutions to fragmentation systems. In particular, it will be shown that there is an explicit, non-trivial, solution that satisfies homogeneneous initial conditions in a pointwise manner. This apparent paradox will be explained in a satisfactory manner by using the theory of Sobolev towers.

If time permits, some brief comments will also be made on the benefits of having an analytic fragmentation semigroup when dealing with related, semilinear coagulation-fragmentation equations.

References

[1] L. Smith, W. Lamb, M. Langer, and A. McBride, *Discrete fragmentation with mass loss*, Journal of Evolution Equations **12** (2012), no. 1, 181–201.

Wha-Suck Lee

Implicit Fokker-Planck Equations: Non-commutative Convolution of Probability Distributions

Feller's operator representation of a single homogenous Markov transition function intertwined by the Chapman Kolmogorov equation is by virtue of the Feller convolution. This convolution is a single state construct and fails to deal with a pair of homogenous Markov transition functions intertwined by an extended Chapman Kolmogorov equation with distinct state spaces. Extended Chapman Kolmogorov equation with distinct state spaces naturally arise in the pair of pseudo Poisson processes of [2, §2] where the distinct state spaces were discrete. The distinctness of the state spaces renders Feller's notion of a simple conditional distribution (mixture) [1, Definition (9.3), V.9] inadequate in describing such pairs. Therefore we introduce a new type of stochastic kernel.

This talk introduces a non-commutative generalization of the Feller convolution in the form of the product of a convolution algebra of admissible homomorphisms to deal with Markov transition functions that arise in the setting of distinct state spaces. This new framework of 'quaternion'-valued admissible homomorphisms on a product test space (i) provides a mathematical interpretation of what Feller created by intuition and (ii) shows that the Fokker-Planck equation for distinct two-state space homogenous processes intertwined by an extended Chapman-Kolmogorov equation is a particular case of an implicit B-evolution equation of empathy theory. Thus, the framework of admissible homomorphisms extends the classical framework of probability distributions.

References

[1] W. Feller, *An introduction to probability theory and its applications, vol ii, second edition*, Wiley Series in Probability and Mathematical Statistics, 1971.

[2] W. Lee and N. Sauer, *Intertwined markov processes: the extended chapman-kolmogorov equation*, Proc. R. Soc. Edinb. A (2017).

M. Lachowicz
University of Warsaw, Poland
H. Leszczyński
University of Gdańsk, Poland
K.A. Topolski
University of Gdańsk, Poland

Self-organization with small range interactions: Creation of bipolarity

We deal with a kinetic equation that may describe the self–organization of various complex systems. We consider the variable interacting rate with small support. This corresponds to interactions of the test entity (individual) with a given internal state only with entities having closed states. We establish all possible equilibrium states and observe the possibility of creation of bipolar (bimodal) distribution that is able to capture interesting behavior in modeling systems, e.g. in political sciences.

Łukasz Leżaj
Wrocław University of Science and Technology, Poland

Heat kernels for subordinators

We study transition densities of subordinators that is Lévy processes in $\mathbb{R}$ starting from 0 with non-decreasing paths. We obtain asymptotic behaviour under lower scaling condition at infinity on the second derivative of the Laplace exponent. Furthermore, we present upper and lower bounds for their density. Sharp estimates are provided if additional upper scaling condition on the Laplace exponent is imposed.

Slawomir Bialecki

Pawel Nalecz-Jawecki

Bogdan Kazmierczak

Tomasz Lipniacki
Institute of Fundamental Technological Research, Warsaw, Poland

From traveling and standing fronts on the curved surfaces to pattern formation

We analyze heteroclinic traveling waves propagating on two dimensional manifolds to show that the modification of the inherent front velocity, in the limit of diffusion tending to zero, is proportional to the geodesic curvature of the front line. As a result, on concave surfaces, stable standing fronts can be formed on lines of constant geodesic curvature. These lines minimize the geometric functional describing the system's energy, consisting of terms proportional to front line-length and to the inclosed surface area. Front pinning at portions of surface with the negative Gaussian curvature provides a mechanism of pattern formation that connects intrinsic surface geometry with the arising pattern, previously characterized by us in 3D case [1]. By considering a system of equations modeling boundary-volume interaction, we show that polarization of the boundary may induce polarization inside the volume. Finally, we provide a link between dynamics of traveling fronts and quantum vortices in superfluids [2], allowing to demonstrate existence of three families of self-similar solutions for front line motion.

References

[1] S. Bialecki, B. Kazmierczak, and T. Lipniacki, *Polarization of concave domains by traveling wave pinning*, PLOS ONE **12** (2017), e0190372.

[2] T. Lipniacki, *Shape-preserving solutions for quantum vortex motion under localized induction approximation*, Physics of Fluids **15** (2003), 1381–1395.

Luca Lorenzi
University of Parma, Italy

On systems of parabolic equations with unbounded coefficients (Part I)

In my talk I will consider systems of parabolic equations coupled up to the first order. The coefficients of the equations will be assumed to be defined in the whole $\mathbb{R}^d$ ($d \geq 1$) and to be possibly unbounded. After a motivation for the study of such systems, I will present some recent results. In particular, I will address the well posedness of the associated Cauchy problem when the initial datum belongs to the space of bounded and continuous functions or to the usual L^p-spaces. Some remarkable properties of the solution of such a Cauchy problem in these two settings of functions, will be also discussed.

Piotr Kalita
Jagiellonian University, Poland
José A. Langa
University of Seville, Spain
Grzegorz Łukaszewicz
University of Warsaw, Poland

Nonlinear semigroups in hydrodynamics and their perturbations. Micropolar meets Newtonian.

We consider the Rayleigh–Bénard problem for two-dimensional Boussinesq systems, for the micropolar fluid and the Navier–Stokes model, respectively. The former model can be regarded as a perturbation of the latter which can be expressed in terms of the associated semigroups.

Our main goal is to compare three important physical characteristics for the both problems, namely, the values of the critical Rayleigh numbers, estimates of the Nusselt numbers and the fractal dimensions of global attractors.

Our estimates reveal the stabilizing effects of micropolarity in comparison with the homogeneous Navier–Stokes fluid.

References:

P. Kalita, J. A. Langa, G. Łukaszewicz, *Micropolar meets Newtonian. The Rayleigh–Bénard problem*, to appear.

Natalia Manakova
South Ural State University, Russia

Optimal Control Problem for the Sobolev Type Equations

A lot of initial-boundary value problems for the equations and the systems of equations not resolved with respect to time derivative are considered in the framework of abstract Sobolev type equations

$$L \dot{x} = Mx + Bu + y, \quad \ker L \neq \{0\}, \tag{1}$$

and

$$L \dot{x} + Mx + N(x) = u, \quad \ker L \neq \{0\}, \tag{2}$$

that make up the vast field of non-classical equations of mathematical physics. The Cauchy problem

$$x(0) - x_0 = 0 \tag{3}$$

for degenerate equations (1) or (2) is unsolvable for arbitrary initial values. We consider the Showalter – Sidorov problem

$$L(x(0) - x_0) = 0 \tag{4}$$

for semi-linear equation (2) and the initial-final problem

$$P_{in}(x(0) - x_0) = 0, \quad P_{fin}(x(0) - x_0) = 0 \tag{5}$$

for linear equation (1), which is a natural generalization of the Cauchy problem.

Generally, the processes applied in mechanics, engineering and production are controllable, therefore, within respective applied problems it is usually essential to control the external actions efficient enough to achieve required results in such processes. Despite the fact that the research field of optimal control problems for distributed systems is rather large, the solutions control matters for confluent semi-linear and linear equations, unresolved for derivative with time, are studied insufficiently. The optimal control problem

$$J(x, u) \to \inf \tag{6}$$

for a Sobolev type equation was first considered by G.A. Sviridyuk and A.A. Efremov. This research provided the basis for a branch of optimal control studies referring to linear and semi-linear Sobolev type equations.

This talk introduces a sufficient conditions for the existence of a solution of the optimal control problems for linear and semi-linear Sobolev type equations. The theory is based on the phase space method. The developed scheme of a numerical method allows one to find an approximate solution to optimal control problems for considered models. On the basis of abstract results the existence of optimal control of processes of filtration and deformation are obtained.

Jan Meichsner
Technische Universität Hamburg, Institut für Mathematik

On the Harmonic Extension Approach to Fractional Powers of Linear Operators

In [2] the authors attracted a lot of attention when describing the action of the fractional Laplacian $(-\Delta)^\alpha$, $\alpha \in (0,1)$, on $f \in L^2(\mathbb{R}^n)$ using a solution u of the ODE

$$u'' + \frac{1-2\alpha}{t} u' = -\Delta u$$

in $L^2(\mathbb{R}^n)$. One has $u(0) = f$ which is why one can consider u as an extension of f and especially in the case of the Laplacian one may interpret u as a harmonic function in "$\mathbb{R}^{n+2-2\alpha}$". Until now many other authors ([1, 3, 4]) contributed by discussing the approach for general sectorial operators A instead of $-\Delta$. Still it is an open problem whether the approach works for all sectorial operators and whether the above introduced solution is actually unique. The talk aims for giving some partial results on these questions.

References

[1] W. Arendt, A. F. M. ter Elst, and M. Warma, *Fractional powers of sectorial operators via the Dirichlet-to-Neumann operator*, Comm. Partial Differential Equations **43** (2018), no. 1, 1–24.

[2] L. Caffarelli and L. Silvestre, *An extension problem related to the fractional Laplacian*, Comm. Partial Differential Equations **32** (2007), no. 8, 1245–1260.

[3] J. E. Galé, P. J. Miana, and P. R. Stinga, *Extension problem and fractional operators: semigroups and wave equations*, Journal of Evolution Equations **13** (2013), no. 2, 343–368.

[4] P. R. Stinga and J. L. Torrea, *Extension Problem and Harnack's Inequality for Some Fractional Operators*, Comm. Partial Differential Equations **35** (2010), no. 11, 2092–2122.

Mustapha Mokhtar-Kharroubi
Université de Bourgogne-Franche Comté, France

Existence of invariant densities for conservative linear kinetic equations on the torus without spectral gaps

This work is the continuation of a general theory, given in *J. Funct. Anal,* **266 (11)** (2014), on time asymptotics of conservative linear kinetic equations on the torus exhibiting a spectral gap. We consider neutron transport-like equations on $L^1(\mathcal{T}^n \times V)$ $(n \geqslant 1)$ where $\mathcal{T}^n := \mathbb{R}^n/(\mathbb{Z})^n$ is the n-dimensional torus under the *conservativity* assumption $\sigma(x,v) = \int_V k(x,v',v)\mu(dv')$ where σ is the collision frequency and k is the scattering kernel while μ is a velocity Radon measure on $\mathbb{R}^n$ with support V. The "collisionless" equation on $\mathcal{T}^n \times V$ is governed by a weighted shift C_0-semigroup $(U(t))_{t\geq 0}$ whose type (or growth bound) $\omega(U) < 0$ *if and only if* there exist $C_1 > 0$ and $C_2 > 0$ such that

$$\int_0^{C_1} \sigma(x+sv,v)ds \geqslant C_2 \text{ a.e. on } \mathcal{T}^n \times V. \tag{1}$$

The full dynamics is governed by a *stochastic* (i.e. mass-preserving on the positive cone) C_0-semigroup $(W(t))_{t\geq 0}$. Under very general assumptions, $(U(t))_{t\geq 0}$ and $(W(t))_{t\geq 0}$ have the same *essential* type $\omega_{ess}(W) = \omega_{ess}(U)$. In particular, under (1) $\omega_{ess}(W) < 0 = \omega(W)$ i.e. $(W(t))_{t\geq 0}$ exhibits a *spectral gap* and 0 is an isolated eigenvalue of $T + K$ with finite algebraic multiplicity. In particular $(W(t))_{t\geq 0}$ tends exponentially to the spectral projection associated to the 0 eigenvalue of the generator. The object of the present work is to consider the critical case $\omega(U) = 0$, i.e. when $\sigma(.,.)$ vanishes on some characteristic curve. In this case, $(W(t))_{t\geq 0}$ has *not* a spectral gap. We provide general tools to study the existence of an invariant density.

Mustapha Mokhtar-Kharroubi
Université de Bourgogne-Franche Comté, France

Relative operator bounds for positive operators in ordered Banach spaces and related topics

It is known that if $A : D(A) \subset X \to X$ is a semibounded (say bounded from above) self-adjoint operator in a Hilbert space X and if $S : D(A) \to X$ is A-bounded then the *relative* A*-bound* of S is equal to

$$\lim_{\lambda \to \infty} \left\| S(\lambda - A)^{-1} \right\|_{\mathcal{L}(X)}.$$

(This result which is still true when A is just α-dissipative *is no longer true outside the realm of Hilbert spaces.*) We show that this result remains true in real ordered Banach spaces X whose *duality map is uniformly monotone on the positive cone* (this covers L^p spaces with $p \geq 2$) if A generates a positive C_0-semigroup on X and S is positive, i.e. $S : D(A) \cap X_+ \to X_+$. This result turns out to be a consequence of *"contractivity on the positive cone"* of $I - C$ for certain positive contractions C. Related results on ergodic projections are also given.

Ref: Contractivity results in ordered spaces. Applications to relative operator bounds and projections with norm one. *Math. Nachr*, 290 (2016).

Marcin Moszyński
University of Warsaw, Poland

Uni-asymptotic linear systems

We consider abstract systems of the form $\{U_s\}_{s\in S}$, where $U_s : X \longrightarrow X$ are bounded linear operators in a norm space X, and S is an arbitrary set. Such a system can be treated, e.g., as formal form of a dynamical system, describing a certain real process as follows: S represents the set of "admissible moments of time", X — the set of "admissible states", and if the initial state of the process is $x \in X$, then $U_s x$ is the state of the process at the time moment $s \in S$.

Our goal is to analyze such systems $\{U_s\}_{s\in S}$, that all their non-trivial trajectories $\{U_s x\}_{s\in S}$ (with $x \neq 0$) "have the same norm-asymptotic behavior" — we call them *uni-asymptotic*. On the other hand, $\{U_s\}_{s\in S}$ is *tight*, when the operator norm and the minimal modulus of U_s "have the same asymptotic behavior".

We prove that uni-asymptoticity is equivalent to tightness if $\dim X < +\infty$. We also prove that the finite dimension is essential above. Some other conditions equivalent to uni-asymptoticity are provided, including asymptotic formulae for the operator norm and for the trajectories, expressed in terms of determinants $\det U_s$.

Delio Mugnolo
University of Hagen, Germany

Comparison principles for parabolic equations and applications to PDEs on networks

If a semi-bounded and symmetric but not essentially self-adjoint operator drives a PDE and boundary conditions have to be imposed and the corresponding solutions can often be compared: an efficient variational principle due to Ouhabaz shows e.g. that the solution to the heat equation with Neumann conditions dominates pointwise and for all times the solution of the heat equation with Dirichlet conditions and same initial data; whereas no such domination can hold if diffusion equations driven by two different elliptic operators under, say, Neumann b.c. are considered.

In this talk I am going to discuss how domination theory can be extended to study domination patterns that only hold on long time scales: I will present (purely spectral!) criteria that imply either "eventual domination" or "interwoven behavior" of orbits of semigroups. Our main application will be given by heat equations on networks: recently obtained results on spectral geometry for quantum graphs turn out to deliver prime examples where such criteria are satisfied.

This is joint work with Gregory Berkolaiko, Jochen Glück, James Kennedy, and Pavel Kurasov.

Felix Ali Mehmeti
University of Valenciennes, France
Kaïs Ammari
University of Monastir, Tunisia
Serge Nicaise
University of Valenciennes, France

Dispersive effects for the Schrödinger equation on graphs

We consider the free Schrödinger equation on graphs with at least one infinite edge. We first describe explictly the kernel of the resolvent operator with an appropriate decomposition corresponding to the discrete spectrum and the continuous one. Then we give sufficient conditions that allow to apply the limit absorbation principle and deduce the resolvent of the identity. Then with an additional assumption and using van der Corput Lemma, the time decay estimates $L^1 \to L^\infty$ in $|t|^{-1/2}$ is proved.

Lassi Paunonen
Tampere University of Technology, Finland

Nonuniform Stability Properties of Coupled Systems

Recent research has demonstrated that polynomial and nonuniform stability of semigroups appear frequently in the study of coupled systems of linear partial differential equations. This is especially the case when PDEs of mixed types are coupled through a shared boundary [3] or inside a common spatial domain [1].

In this presentation we consider a selection of such PDE systems and discuss how many of them can be viewed as particular cases of composite systems consisting of an unstable and a stable abstract linear differential equation. As our main results we present new general conditions for proving the polynomial or nonuniform stability of coupled PDE systems in this abstract setting [2]. The results are motivated by and primarily applicable for coupled PDEs on one-dimensional or geometrically simple spatial domains. We also discuss the degrees of stability given by the abstract results compared to those obtained by direct approaches.

References

[1] E. M. Ait Ben Hassi, K. Ammari, S. Boulite, and L. Maniar, *Stability of abstract thermo-elastic semigroups*, J. Math. Anal. Appl. **435** (2016), no. 2, 1021–1035.

[2] Lassi Paunonen, *Stability and Robust Regulation of Passive Linear Systems*, ArXiv e-prints (https://arxiv.org/abs/1706.03224) (2017).

[3] Xu Zhang and Enrique Zuazua, *Long-time behavior of a coupled heat-wave system arising in fluid-structure interaction*, Arch. Ration. Mech. Anal. **184** (2007), no. 1, 49–120.

Sergey Piskarev
Lomonosov Moscow State University, Russia

Approximation of fractional differential equations in Banach spaces

In this talk, we present our research on convergence of difference schemes for fractional differential equations. Using implicit difference scheme and explicit difference scheme, we have a deal with the full discretization (in space and in time) of the solutions of fractional differential equations in Banach spaces. A lot of papers were devoted to the discretization of C_0-semigroups in traditional way. Recently, we considered the discrete approximation of integrated semigroups [1], where the order of convergence was obtained using ill-posed problems theory. In this talk we continue our investigations [2] on discretization of differential equations of fractional order $0 < \alpha < 1$ in Banach spaces and get the optimal [3] order of convergence $O(\tau_n^\alpha)$.

We develop such approach and consider approximation of semilinear equations in the form

$$D^\alpha u(t) = Au(t) + f(u(t)),\ u(0) = u^0,$$

where $D^\alpha u(t)$ is a fractional derivative of $u(t)$ in Caputo sense and function $f(\cdot)$ is smooth enough. The approximation of such problems is based on the principle of compact approximation.

References

[1] V. Morozov M. Li and S. Piskarev, *On the approximations of derivatives of integrated semigroups*, J. Inverse Ill-Posed Probl. **18** (2010), no. 5, 515–550.

[2] M. Li R. Liu and S. Piskarev, *Stability of difference schemes for fractional equations*, Differ. Equ. **51** (2015), no. 7, 904–924.

[3] ———, *The order of convergence of difference schemes for fractional equations*, Numerical Functional Analysis and Optimization **38** (2017), no. 6, 754–769.

Jacek Polewczak
California State University, USA

Hard-spheres linear kinetic theories

One of the important features of any kinetic theory is to provide theoretical predictions of transport coefficients. From the mathematical point of view, this problem is intrinsically related to the study of the spectrum of the corresponding linearized kinetic equation (linearized about an absolute maxwellian). Knowledge of the spectrum of the linearized Boltzmann operator is rather complete. This is not the case for the hard-spheres linear kinetic theory based on the revised Enskog equation. I will discuss the spectrum of the linearized revised Enskog operator, including the connections to hydrodynamic descriptions of a fluid.

References

[1] H. van Beijeren and M. H. Ernst, *The Modified Enskog Equation*, Physica, **68** (1973), 437-456.

[2] H. van Beijeren and M. H. Ernst, *The Modified Enskog Equation for Mixtures*, Physica, **70** (1973), 225-242.

[3] P. Resibois, *The Linearized (Modified) Enskog Equation: The Approach to Equilibrium*, Physica, **94A** (1978), 1-19.

[4] I. M. de Schepper and E. G. D. Cohen, *Very-Short-Wavelength Collective Modes in Fluids*, Journal of Statistical Physics, **27**(2) (1982), 223-281.

[5] M. Lachowicz, *Is the Linearized Boltzmann-Enskog Operator Dissipative?*, Applied Mathematics Letters **14** (2001), 291-295.

Jacek Banasiak
University of Pretoria, South Africa
Aleksandra Puchalska
University of Warsaw, Poland

The graph structure impact on a singular limit of the generalized network transport

In the talk we present the family of perturbed transport problems in which a domain consists of countable intervals coupled by transition conditions at the ends. Using the theory of convergence of sequences of semigroups, we present different convergence results in the case of velocities of transport that accelerates to infinity being balanced by certain conditions at the boundary. We compare the structure of graph in primal problems with the properties of a network of the limit solution.

Abdelaziz Rhandi
University of Salerno, Italy

Weighted Hardy's inequalities and Kolmogorov-type operators

We give general conditions to state the weighted Hardy inequality

$$c\int_{\mathbb{R}^N}\frac{\varphi^2}{|x|^2}d\mu \leq \int_{\mathbb{R}^N}|\nabla\varphi|^2 d\mu + C\int_{\mathbb{R}^N}\varphi^2 d\mu, \quad \varphi\in C_c^\infty(\mathbb{R}^N),\ c\leq c_{0,\mu},$$

with respect to a probability measure $d\mu$. Moreover, the optimality of the constant $c_{0,\mu}$ is given. The inequality is related to the following Kolmogorov equation perturbed by a singular potential

$$Lu + Vu = \left(\Delta u + \frac{\nabla\mu}{\mu}\cdot\nabla u\right) + \frac{c}{|x|^2}u$$

for which the existence of positive solutions to the corresponding parabolic problem can be investigated. The hypotheses on $d\mu$ allow the drift term to be of type $\frac{\nabla\mu}{\mu} = -|x|^{m-2}x$ with $m > 0$.

Joint work with A. Canale, F. Gregorio and C. Tacelli.

Quentin Richard
University of Bourgogne Franche-Comté, France

Time asymptotics of structured populations with diffusion

In this talk we will study the following size-structured population model [1]

$$\begin{aligned} \partial_t u(t,s) + \partial_s(\gamma(s)u(t,s)) = \partial_s(d(s)\partial_s(u(t,s))) \quad &- \quad \mu(s)u(t,s) \\ &+ \quad \int_0^m \beta(s,y)u(t,y)\mathrm{d}y, \end{aligned}$$

for $s \in [0,m]$ and $m < \infty$. We consider some Feller boundary conditions and we verify that the problem is well-posed in the sense of the semigroups theory in $L^1(0,m) \times \mathbb{R}^2$. With Hopf maximum principle we prove that the semigroup is irreducible. Using weak compactness arguments, we show first a stability result of the essential type then deduce that the semigroup has a spectral gap and consequently the asynchronous exponential growth property. Finally [2] we show how to extend this theory to models with arbitrary sizes and give a theoretical condition to get the asynchronous behavior.

References

[1] J.Z. Farkas and P. Hinow, *Physiologically structured populations with diffusion and dynamic boundary conditions*, Mathematical Biosciences and Engineering (2011), 503–513.

[2] M. Mokhtar-Kharroubi and Q. Richard, *Time asymptotics of structured populations with diffusion and dynamic boundary conditions*, to appear in Discrete and Continuous Dynamical Systems - series B.

Massimiliano Rosini
Maria Curie-Skłodowska University, Poland

On the micro-to-macro limit for 1D scalar conservation laws

We provide an overview of results on the derivation of 1D scalar conservation laws via ODEs systems of deterministic particles interacting via follow-the-leader interactions. The main motivation behind this problem arises in traffic flow modelling. We present results on the derivation of entropy solutions of the Cauchy problem of the LWR model [1, 3] and later extensions of this result on problems with Dirichlet boundary data [2] and on similar models such as the ARZ model [4] for traffic flow and the Hughes model [5] for pedestrians. The results are joint with S. Fagioli, M. Di Francesco and G. Russo.

References

[1] M. Di Francesco, S. Fagioli, and M. D. Rosini, *Deterministic particle approximation of scalar conservation laws*, Boll. Unione Mat. Ital. **10** (2017), no. 3, 487–501. MR 3691810

[2] M. Di Francesco, S. Fagioli, M. D. Rosini, and G. Russo, *Follow-the-leader approximations of macroscopic models for vehicular and pedestrian flows*, Active particles. Vol. 1. Advances in theory, models, and applications, Model. Simul. Sci. Eng. Technol., Birkhäuser/Springer, Cham, 2017, pp. 333–378. MR 3644595

[3] M. Di Francesco and M. D. Rosini, *Rigorous derivation of nonlinear scalar conservation laws from follow-the-leader type models via many particle limit*, Arch. Ration. Mech. Anal. **217** (2015), no. 3, 831–871. MR 3356989

[4] Marco di Francesco, Simone Fagioli, and Massimiliano D. Rosini, *Many particle approximation of the Aw-Rascle-Zhang second order model for vehicular traffic*, Math. Biosci. Eng. **14** (2017), no. 1, 127–141. MR 3562902

[5] Marco Di Francesco, Simone Fagioli, Massimiliano Daniele Rosini, and Giovanni Russo, *Deterministic particle approximation of the Hughes model in one space dimension*, Kinet. Relat. Models **10** (2017), no. 1, 215–237. MR 3579570

Jan Rozendaal
Polish Academy of Sciences, Poland
Australian National University, Australia

Sharp growth rates for semigroups using resolvent bounds

In this talk I will discuss some recent results from [1], concerning growth rates for strongly continuous semigroups. These results show that a growth rate for the resolvent bounds of a semigroup generator on imaginary lines implies a corresponding growth rate for the semigroup $(T(t))_{t\geq 0}$ if either the underlying space X is a Hilbert space, or the semigroup is asymptotically analytic, or if the semigroup is positive and X is an L^p-space or a space of continuous functions. I will also discuss variations of the main results on fractional domains; these are valid on more general Banach spaces. Finally, the main theorem can be applied to obtain optimality in a classical example by Renardy of a perturbed wave equation with unusual spectral behavior.

This is joint work with Mark Veraar (Delft University of Technology).

References

[1] J. Rozendaal and M. Veraar, *Sharp growth rates for semigroups using resolvent bounds*, Online at `https://arxiv.org/abs/1712.00692`, 2017.

Ryszard Rudnicki
Polish Academy of Sciences, Poland

Stochastic semigroups and their applications to Stein's neural model

We start with some results concerning asymptotic decomposition of substochastic operators and semigroups [1, 2] Then we present corollaries useful in studying of piecewise deterministic Markov processes [4]. We recall Stein's description of activity of neuronal cells and we take notice of problems which appear in its formulation in terms of stochastic semigroups. Finally, we apply results concerning stochastic semigroups to this model [3].

References

[1] K. Pichór and R. Rudnicki, *Asymptotic decomposition of substochastic operators and semigroups*, J. Math. Anal. Appl. **436** (2016), 305–321.

[2] ———, *Asymptotic decomposition of substochastic semigroups and applications*, Stochastics and Dynamics **18** (2018), 1850001–18.

[3] ———, *Stability of stochastic semigroups and applications to stein's neuronal model*, Discrete Contin. Dyn. Syst. B **23** (2018), 377–385.

[4] R. Rudnicki and M.Tyran-Kamińska, *Piecewise deterministic processes in biological models*, SpringerBriefs in Mathematical Methods, Springer, Cham, Switzerland, 2017.

Wolfgang M. Ruess
University Duisburg-Essen, Germany

Regularity of solutions to partial differential delay equations

In the context of the partial differential delay equation $\dot{u}(t) + Bu(t) = F(u_t)$ for $t \geq 0$, with $u_{|I} = \varphi \in E = BUC(I, X)$, with $B : D(B) \subset X \to X$ linear $\omega-$m-accretive, X Banach, $F : \hat{E} \subset E \to X$ (history control), and $I \in \{[-R, 0], (-\infty, 0]\}$, we consider the problem under which 'smoothness' assumptions on the history-operator F and the initial history φ a) the mild solution to the equation is actually a classical solution, and b) its time-derivative is a mild solution to a linearisation of the original equation. The proof of the result is carried out by inspecting differentiability of (a motion of) the (nonlinear) solution semigroup associated to the equation in the initial history space E. Typical applications for the abstract equation are models of population dynamics.

Artur Rutkowski
Wrocław University of Science and Technology, Poland

The Dirichlet problem for nonlocal Lévy-type operators

We consider nonlocal operators of the form

$$Lu(x) = PV \int_{\mathbb{R}^d} (u(x) - u(x+y))\nu(dy),$$

where ν is a Lévy measure. Such operators are the generators of the semigroup corresponding to a Lévy process with jump intensity ν.

For symmetric Lévy measures ν, and bounded open sets D we show that the Dirichlet problem

$$\begin{cases} Lu = f & \text{in } D, \\ u = g & \text{on } D^c, \end{cases}$$

has a weak solution whenever $f \in L^2(D)$ and g can be extended to a function from a certain Sobolev space.

We also discuss, under some assumptions, the sufficient and necessary conditions under which g possesses such an extension.

The talk is based on joint works with Krzysztof Bogdan, Tomasz Grzywny, and Katarzyna Pietruska-Pałuba [1, 2, 3].

References

[1] Krzysztof Bogdan, Tomasz Grzywny, Katarzyna Pietruska-Pałuba, and Artur Rutkowski, *Extension theorem for nonlocal operators*, ArXiv e-prints (2017).

[2] Krzysztof Bogdan and Artur Rutkowski, *Trace and regularity for nonlocal Sobolev spaces*, 2018.

[3] Artur Rutkowski, *The Dirichlet problem for nonlocal Lévy-type operators*, Publicacions Matemàtiques **62** (2018), no. 1, 213–251.

Christian Seifert
Hamburg University of Technology, Germany

Perturbations of positive semigroups with applications to Dirichlet forms perturbed by jump parts

In [1] the authors proved the upper bound

$$p_t(x,y) \leq p_t^0(x,y) + t\|j\|_\infty \tag{1}$$

for the heat kernel for jump processes by means of splitting the jump density into a bounded part called j and the remaining part, whose heat kernel is $(p_t^0)_t$. This procedure is sometimes called Meyer decomposition. The talk will describe the functional analytic variant of the result, namely a perturbation result for positive C_0-semigroups on L_p-spaces, where the perturbation may act on a different L_q-space. In the special case of Dirichlet forms perturbed by jump parts with bounded jump density we thus recover (1), however the semigroup approach let us allow for more general jump densities. It turns out that the perturbation result is essentially a consequence of positivity, so the contractivity properties of the submarkovian semigroups associated with the Dirichlet forms do not play a role here.

The talk is based on joint works with H. Vogt, M. Waurick and D. Wingert [2, 3, 4].

References

[1] M.T. Barlow, A. Grigor'yan, and T. Kumagai, *Heat kernel upper bounds for jump processes and the first exit time*, J. Reine Angew. Math. **626** (2009), 135–157.

[2] C. Seifert, H. Vogt, and M. Waurick, *A note on perturbations of C_0-semigroups*, arXiv Preprint: 1802.00335, 2018.

[3] C. Seifert and M. Waurick, *Perturbations of positive semigroups on L_p-spaces*, Positivity **20** (2016), no. 2, 467–481.

[4] C. Seifert and D. Wingert, *On the perturbation of positive semigroups*, Semigroup Forum **91** (2015), no. 2, 495–501.

Christian Seifert
Hamburg University of Technology, Germany

The linearized KdV equation on metric graphs

The Korteweg-de Vries equation

$$\partial_t u = \partial_x^3 u + c\partial_x u - 6u\partial_x u$$

with some constant $c \in \mathbb{R}$ describes waves in shallow water channels. Linearizing around the stationary solution $u = 0$ yields the so-called Airy equation, which is of the form

$$\partial_t u = \alpha\partial_x^3 u + \beta\partial_x u,$$

where α, β are constants. We will study this linearized KdV equation on metric graphs by means of semigroup methods. The main aim is to characterize coupling conditions at the vertices of the metric graph such that the equation becomes well-posed and has "nice" properties.

This talk is based on a joint work with D. Mugnolo and D. Noja, see [1], [2].

References

[1] D. Mugnolo, D. Noja, and C. Seifert, *Airy-type evolution equations on star graphs*, Anal. PDE **11** (2018), no. 7, 1625–1652.

[2] C. Seifert, *The linearised Korteweg-de Vries equation on general metric graphs*, The Diversity and Beauty of Applied Operator Theory (A. Böttcher, D. Potts, P. Stollmann, and D. Wenzel, eds.), Operator Theory: Advances and Applications 268, Birkhäuser, 2018.

David Seifert
University of Oxford, United Kingdom

Optimal rates of decay for semigroups on Hilbert spaces

We discuss the quantitative asymptotic behaviour of operator semigroups. In [2] Batty and Duyckaerts obtained upper and lower bounds on the rate of decay of a semigroup given bounds on the resolvent growth of the semigroup generator. They conjectured that in the Hilbert space setting and for the special case of polynomial resolvent growth it is possible to improve the upper bound so as to yield the exact rate of decay up to constants. This conjecture was proved to be correct by Borichev and Tomilov in [3], and the conclusion was extended by Batty, Chill and Tomilov in [1] to certain cases in which the resolvent growth is not exactly polynomial but almost. In this talk we extend their result by showing that one can improve the upper bound under a significantly milder assumption on the resolvent growth. This result is optimal in a certain sense. We also discuss how this improved result can be used to obtain sharper estimates on the rate of energy decay for a wave equation subject to viscoelastic damping at the boundary. The talk is based on joint work with J. Rozendaal and R. Stahn.

References

[1] C.J.K. Batty, R. Chill, and Y. Tomilov, *Fine scales of decay of operator semigroups*, J. Eur. Math. Soc. **18** (2016), no. 4, 853–929.

[2] C.J.K. Batty and T. Duyckaerts, *Non-uniform stability for bounded semigroups on banach spaces*, J. Evol. Equ. **8** (2008), no. 4, 765–780.

[3] A. Borichev and Y. Tomilov, *Optimal polynomial decay of functions and operator semigroups*, Math. Ann. **347** (2010), no. 2, 455–478.

Sophiya A. Zagrebina
South Ural State University, Russia
Natalya N. Solovyova
South Ural State University, Russia

Nonclassical Conditions for Linear Sobolev Type Equations

Mathematical model whose prototypes are Barenblatt–Zheltov–Kochina equation and Hoff equation has been considered in the sequence spaces which are analogues of Sobolev function spaces.

The peculiarities of our approach are, firstly, the active use of the theory of bounded operators and the degenerate holomorphic groups of operators generated by them. Secondly, it is also the theory of positive groups of operators, defined on Banach lattices, to lay the basis of the theory of positive degenerate holomorphic groups of operators which phase spaces are Banach lattices. Thirdly, we research the concrete mathematical model which has been considered in Sobolev sequence spaces, that can be interpreted as the space of Fourier coefficients of solutions of initial-boundary value problems for Barenblatt–Zheltov–Kochina equation or Hoff equation.

The sufficient conditions have been described for positive solution in this mathematical model. It is noted that the Barenblatt–Zheltov–Kochina equation satisfies the found sufficient conditions, and therefore the initial-boundary value problem can have non-negative solutions. The basis of our research is the theory of the positive semigroups of operators and the theory of degenerate holomorphic groups of operators. As a result of the merging of these theories a new theory of degenerate positive holomorphic groups of operators has been obtained. At present, the main results have already been obtained, based not on the Ito–Stratonovich–Skorokhod approach, but on the Nelson–Glickich derivative. Further consider both various generalizations of the Showalter–Sidorov condition and the relation between the powers of the polinomials of operators. The results of a new theory can be applied in economic and engineering problems.

Chiara Spina
University of Salento, Italy

Joint work with **Giorgio Metafune, Luigi Negro** and **Motohoro Sobajima.**

Rellich and Calderón-Zygmund inequalities for operators with discontinuous and singular coefficients

We give necessary and sufficient conditions for the validity of Rellich and Calderón-Zygmund inequalities in $L^p(\mathbb{R}^N)$ and in $L^p(B)$, where B is the unitary ball of $\mathbb{R}^N$, for the operator

$$L = \Delta + (a-1)\sum_{i,j=1}^{N} \frac{x_i x_j}{|x|^2} D_{ij} + c\frac{x}{|x|^2}\cdot\nabla - b|x|^{-2},$$

with $a > 0$, $b, c \in \mathbb{R}$. Note that the condition $a > 0$ is equivalent to the ellipticity of the principal part of L.
More specifically, in the case of $L^p(\mathbb{R}^N)$, set

$$\gamma_p(a,c) := \left(\frac{N}{p} - 2\right)\left(N - 1 + c + a\Big(1 - \frac{N}{p}\Big)\right)$$

and considered the parabola

$$P_{p,a,c} := \left\{\lambda = -a\xi^2 + i\xi\left(N - 1 + c + a\left(3 - \frac{2N}{p}\right)\right) - \gamma_p(a,c)\ ;\ \xi \in \mathbb{R}\right\},$$

we prove that for $N \geq 2,\ a > 0,\ b,\ c \in \mathbb{R},\ 1 \leq p \leq \infty$, there exists a positive constant $C = C(N, a, p, c, b)$ such that the inequality

$$\int_{\mathbb{R}^N} |x|^{\alpha p}|Lu|^p\,dx \geq C\int_{\mathbb{R}^N} |x|^{p(\alpha-2)}|u|^p\,dx\|Lu\|_p \geq C\||x|^{-2}|u|\|_p$$

holds for every $u \in C_c^\infty(\mathbb{R}^N \setminus \{0\})$, if and only if $b + \lambda_j \notin P_{p,a,c}$ for every $j \in \mathbb{N}$, where λ_j are the eigenvalues of Laplace-Beltrami operator. The situation is more involved for the bounded domain B. However, also in this case, we give necessary and sufficient conditions for the validity of Rellich inequalities. As work in progress, we are investigating the validity of Rellich type inequalities with some correction terms in the cases where they fail in the form above written. Some partial results have been obtained.

Moreover we find necessary and sufficient conditions for the validity of the estimate

$$\int_{\mathbb{R}^N} |D^2u|^p\,dx \leq C\int_{\mathbb{R}^N} |Lu|^p\,dx,$$

where $u \in W^{2,p}(\mathbb{R}^N)$, $1 < p < \infty$.

We point out that Rellich and Calderón-Zygmund inequalities have been widely studied in literature for the Laplacian and more general operators. Firstly Rellich inequalities for the Laplacian in L^2 spaces (according to our notations $a=1,\ b=c=0$)

$$\left(\frac{N(N-4)}{4}\right)^2 \int_{\mathbb{R}^N} |x|^{-4}|u|^2\,dx \le \int_{\mathbb{R}^N} |\Delta u|^2\,dx,$$

for $N \neq 2$ and for every $u \in C_c^\infty(\mathbb{R}^N \setminus \{0\})$, have been proved by Rellich in 1956 and then extended to L^p-norms for $1 < p < \frac{N}{2}$.
Rellich inequalities with respect to the weight $|x|^\alpha$ have been also studied.

Georgy Sviridyuk
South Ural State University, Russia

Sobolev Type Equations. Degenerate Semigroups of Operators and Degenerate Propagators

Sobolev type equations were firstly studied in the works of A. Poincare. Then they appeared in the works of S.V. Oseen, J.V. Boussinesq, S.G. Rossby and other researchers, that were dedicated to the investigation of some hydrodynamics problems. Their systematical study started in the middle of the XX century with the works of S.L. Sobolev. The first monograph devoted to the study of such equations appeared in 1999. Nowadays the number of works devoted to such equations is increasing extensively. Sometimes such equations are called equations that are not of Cauchy–Kovalevskaya type, pseudoparabolic equations, degenerate equations or equations unsolvable with respect to the highest derivative. The term "Sobolev type equations" was firstly proposed in the works of R. Showalter. Nowadays Sobolev type equations constitute the vast area in nonclassical equations of mathematical physics. The proposed theory of degenerate semigroups of operators is a suitable mathematical tool for the study of such equations. The theory is developing in different directions: optimal control problems, initial-final value problems, equations of high order, and finds applications in elasticity theory, fluid dynamics, oil production, economics, biology, and in the solution of many technical problems, for example in the theory of dynamic measurements.

Karol Szczypkowski
Wrocław University of Science and Technology, Poland

Fundamental solution for super-critical non-symmetric Lévy-type operators

In the talk I will present the results of the paper [4], which is a sequel to [2]. The aim is to construct the fundamental solution p^κ to the equation $\partial_t = \mathcal{L}^\kappa$, where under certain assumptions the operator $\mathcal{L}^\kappa$ takes the form,

$$\mathcal{L}^\kappa f(x) := \int_{\mathbb{R}^d} (f(x+z) - f(x) - 1_{|z|<1} \langle z, \nabla f(x) \rangle) \kappa(x,z) J(z)\, dz\,.$$

In particular, $J\colon \mathbb{R}^d \to [0,\infty]$ is a Lévy density, i.e., $\int_{\mathbb{R}^d} (1 \wedge |x|^2) J(x) dx < \infty$. The function $\kappa(x,z)$ is assumed to be Borel measurable on $\mathbb{R}^d \times \mathbb{R}^d$ satisfying $0 < \kappa_0 \le \kappa(x,z) \le \kappa_1$, and $|\kappa(x,z) - \kappa(y,z)| \le \kappa_2 |x-y|^\beta$ for some $\beta \in (0,1)$.

We concentrate on the case when the order of the operator is positive and smaller or equal 1 (without excluding higher orders up to 2). The lack of the symmetry of the Lévy density $\kappa(x,z)J(z)$ in z variable may cause a non-zero *internal drift*, which reveals itself as a gradient term in the decomposition

$$\begin{aligned}\mathcal{L}^\kappa f(x) = &\int_{\mathbb{R}^d} (f(x+z) - f(x) - 1_{|z|<r} \langle z, \nabla f(x) \rangle)\, \kappa(x,z) J(z) dz \\ &+ \left(\int_{\mathbb{R}^d} z \left(1_{|z|<r} - 1_{|z|<1} \right) \kappa(x,z) J(z) dz \right) \cdot \nabla f(x)\,.\end{aligned}$$

Our approach rests on imposing conditions on the expression

$$\int_{r \le |z| < 1} z\, \kappa(x,z) J(z) dz.$$

We prove the uniqueness, estimates, regularity and other qualitative properties of p^κ. The result is new even for 1-stable Lévy measure $J(z) = |z|^{-d-1}$, cf. [3] and [1].

References

[1] Zhen-Qing Chen and Xicheng Zhang, *Heat kernels for time-dependent non-symmetric stable-like operators*, preprint 2017, arXiv:1709.04614.

[2] Tomasz Grzywny and Karol Szczypkowski, *Heat kernels of non-symmetric Lévy-type operators*, preprint 2018, arXiv:1804.01313.

[3] P. Jin, *Heat kernel estimates for non-symmetric stable-like processes*, preprint 2017, arXiv:1709.02836.

[4] Karol Szczypkowski, *Fundamental solution for super-critical non-symmetric Lévy-type operators*, preprint 2018, arXiv:1807.04257.

Cristian Tacelli
University of Salerno

Elliptic operators with unbounded diffusion, drift and potential terms

We prove that the realization A_p in $L^p(\mathbb{R}^N)$, $1 < p < \infty$, of the elliptic operator $A = (1+|x|^\alpha)\Delta + b|x|^{\alpha-1}\frac{x}{|x|}\cdot\nabla - c|x|^\beta$ with domain $D(A_p) = \{u \in W^{2,p}(\mathbb{R}^N) \mid Au \in L^p(\mathbb{R}^N)\}$ generates a strongly continuous analytic semigroup $T(\cdot)$ provided that $\alpha > 2$, $\beta > \alpha - 2$ and any constants $b \in \mathbb{R}$ and $c > 0$. This generalizes the recent results in [1] and in [2].

Moreover we prove that the heat kernel k associated to A satisfies

$$k(t,x,y) \leq c_1 e^{\lambda_0 t + c_2 t^{-\gamma}} \left(\frac{1+|y|^\alpha}{1+|x|^\alpha}\right)^{\frac{b}{2\alpha}} \frac{(|x||y|)^{-\frac{N-1}{2}-\frac{1}{4}(\beta-\alpha)}}{1+|y|^\alpha} e^{-\frac{\sqrt{2}}{\beta-\alpha+2}\left(|x|^{\frac{\beta-\alpha+2}{2}}+|y|^{\frac{\beta-\alpha+2}{2}}\right)}$$

for $t > 0$, $|x|$, $|y| \geq 1$, where $b \in \mathbb{R}$, c_1, c_2 are positive constants, λ_0 is the largest eigenvalue of the operator A, and $\gamma = \frac{\beta-\alpha+2}{\beta+\alpha-2}$. The proof is based on the relationship between the log-Sobolev inequality and the ultracontractivity of a suitable semigroup in a weighted space.

Joint work with S.E. Boutiah, F. Gregorio and A. Rhandi.

References

[1] A. Canale, A. Rhandi, and C. Tacelli, *Schrödinger type operators with unbounded diffusion and potential terms*, Ann. Sc. Norm. Super. Pisa Cl. Sci. **XVI** (2016), no. 2, 581–601.

[2] G. Metafune, C. Spina, and C. Tacelli, *Elliptic operators with unbounded diffusion and drift coefficients in L^p spaces*, Adv. Diff. Equat **19** (2012), no. 5-6, 473–526.

Andrzej Tomski
University of Silesia in Katowice, Poland

Joint work with **Marta Tyran-Kamińska, Michael Mackey** and **Paweł Klimasara.**

Semigroups in biophysics: stochastic Liouville equation

Long time behaviour problem in the model of stochastic gene expression, where we consider one specific gene (or just a particle) is known and solved [2]. Stochastic semigroups play important role in mathematical description of such phenomenon. Here we will discuss new concept introduced by American physicist Paul Bressloff [1] to investigate the influence of the stochastic environmental input to the whole population of particles. It appears that stochastic semigroups can be used in mathematical description of the problem once again.

References

[1] P. Bressloff, *Stochastic Liouville equation for particles driven by dichotomous environmental noise.*, Physical Review E **95** (2017), no. 012124.

[2] R. Rudnicki and A. Tomski, *On a stochastic gene expression with pre-mRNA, mRNA and protein contribution*, Journal of Theoretical Biology **21** (2015), no. 387, 54–67.

Roberto Triggiani
University of Memphis, USA
University of Virginia, USA

A third order (in time) PDE: a view from the boundary, to control and to observe

We shall consider a third order (in time) PDE that arises in many applications in different fields. The emphasis is a study of its behavior from the boundary of the bounded domain in which it evolves: optimal interior and boundary regularity with Dirichlet or Neumann non-homogeneous term (control); corresponding boundary control problems; inverse problems with boundary observation. Thus, the forcing term/observation is non-invasive. Different problems dictate different math techniques: semigroups methods for homogeneous BC; energy methods (differential in the non-homogeneous Dirichlet boundary case and pseudo-differential/microlocal analysis in the non-homogeneous Neumann boundary case) for optimal interior and boundary regularity; Carleman-type estimates for boundary control and inverse problems from the boundary.

Sascha Trostorff
Technical University of Dresden, Germany

Strongly continuous semigroups associated with evolutionary equations

Evolutionary equations, as they were introduced in [1], are abstract operator equations of the form

$$(\partial_t \mathcal{M} + \mathcal{A})u = f,$$

where ∂_t denotes the derivative with respect to time, $\mathcal{M}$ is a bounded linear space-time operator and $\mathcal{A}$ is an unbounded spatial operator. In contrast to classical evolution equations, evolutionary equations are formulated on the whole real line as time horizon and so, no explicit initial condition is needed to complete the equation.

Within the framework of evolutionary equations we provide a way to incorporate initial value problems or, more generally, problems with prescribed history. The given initial data/history will enter the equation as a distributional right hand side. The main problem is to determine those data, which allow for a continuous solution. We define a space of admissible histories which allows us to associate a strongly continuous semigroup with the given evolutionary problem. The presented results were obtained in [2].

References

[1] R. Picard, *A structural observation for linear material laws in classical mathematical physics*, Math. Methods Appl. Sci. **32** (2009), no. 14, 1768–1803.

[2] S. Trostorff, *Exponential stability and initial value problems for evolutionary equations*, Habilitation thesis, TU Dresden, 2018.

Marta Tyran-Kamińska
Polish Academy of Sciences, Poland
University of Silesia in Katowice, Poland

Substochastic semigroups and positive perturbations of boundary conditions

We study well-posedness of linear evolution equations on L^1 of the form

$$u'(t) = Au(t), \quad \Psi_0 u(t) = \Psi u(t), \quad t > 0, \quad u(0) = f, \tag{1}$$

where Ψ_0, Ψ are positive unbounded linear operators and the linear operator A is such that equation (1) with $\Psi = 0$ generates a substochastic semigroup on L^1. We provide sufficient conditions for the operator A to be the generator of a positive semigroup as well as of a stochastic semigroup. This extends the approach of Greiner [1] by considering unbounded Ψ and positive semigroups. We also show how to obtain stationary solutions of (1). We illustrate our results with a two-phase age-size-dependent cell cycle model given by a piecewise deterministic Markov process.

This talk is based on a joint work with P. Gwiżdż [2, 3].

References

[1] Günther Greiner, *Perturbing the boundary conditions of a generator*, Houston J. Math. **13** (1987), no. 2, 213–229. MR 904952

[2] Piotr Gwiżdż and Marta Tyran-Kamińska, *Densities for piecewise deterministic Markov processes with boundary*, Preprint (2018).

[3] ———, *Positive semigroups and perturbations of boundary conditions*, Preprint (2018).

Adam Skalski
Polish Academy of Sciences, Poland
Ami Viselter
University of Haifa, Israel

Convolution semigroups on quantum groups and non-commutative Dirichlet forms

We will discuss convolution semigroups of states on locally compact quantum groups. They generalize the families of distributions of Lévy processes from probability. We are particularly interested in semigroups that are symmetric in a suitable sense. These are proved to be in one-to-one correspondence with KMS-symmetric Markov semigroups on the L^∞ algebra that satisfy a natural commutation condition, as well as with non-commutative Dirichlet forms on the L^2 space that satisfy a natural translation invariance condition. This Dirichlet forms machinery turns out to be a powerful tool for analyzing convolution semigroups as well as proving their existence. We will use it to derive geometric characterizations of the Haagerup Property and of Property (T) for locally compact quantum groups, unifying and extending earlier partial results. If time permits, we will also show how examples of convolution semigroups can be obtained via a cocycle twisting procedure. Based on [1].

References

[1] A. Skalski and A. Viselter, *Convolution semigroups on locally compact quantum groups and noncommutative Dirichlet forms*, J. Math. Pures Appl., to appear, doi: 10.1016/j.matpur.2018.04.007.

Hendrik Vogt
University of Bremen, Germany

Perturbation theory for accretive operators in L_p

We present a perturbation theorem for m-accretive operators in L_p, where $1 < p < \infty$, that resembles the KLMN theorem for perturbation of self-adjoint operator in Hilbert space. Clearly, the advantage of the new theorem is that one is no longer restricted to the case $p = 2$. The drawback is that the theorem only deals with perturbation by multiplication operators.

We explain how the result can be used to associate a quasi-m-accretive operator in $L_p(\Omega)$ with the formal differential expression

$$\mathcal{L} = -\nabla \cdot (a\nabla) + b_1 \cdot \nabla + \nabla \cdot b_2 + Q$$

on an open set $\Omega \subseteq \mathbb{R}^N$, with *complex* measurable coefficients $a \colon \Omega \to \mathbb{C}^{N\times N}$, $b_1, b_2 \colon \Omega \to \mathbb{C}^N$ und $Q \colon \Omega \to \mathbb{C}$.

Jürgen Voigt
Technical University of Dresden, Germany

On “monotone” convergence of sectorial forms

Let H be a complex Hilbert space, and let (a_n) be a sequence of closed sectorial forms. Suppose that (a_n) is “increasing” in the sense that the sequence $(\mathrm{dom}(a_n))$ is decreasing, and that there exists $\theta \in [0, \pi/2)$ such that $a_{n+1} - a_n$ is sectorial with vertex 0 and angle θ for all $n \in \mathbb{N}$. Then there exists a limiting form a such that the sequence (A_n) of operators A_n associated with a_n converges to the operator A associated with a, in the strong resolvent sense. This result – a generalisation of [3, Theorems 3.1 and 4.1] – is due to Batty and ter Elst [1]. We present a proof along the lines of a proof given in [2, proof of Theorem 5] and also show that Ouhabaz’ result follows from [1].

The talk is based on joint work with H. Vogt.

References

[1] C. J. K. Batty and A. F. M. ter Elst, *On series of sectorial forms*, J. Evol. Equ. **14** (2014), 29–47.

[2] E.-M. Ouhabaz, *Second order elliptic operators with essential spectrum* $[0, \infty)$ *on* L^p, Comm. Partial Differential Equations **20** (1995), 763–773.

[3] B. Simon, *A canonical decomposition for quadratic forms with applications to monotone convergence theorems*, J. Funct. Anal. **28** (1978), 377–385.

Radosław Wieczorek
University of Silesia, Poland

Clustering in a model of yeast cell cycle

Autonomous oscillation and clustering of cell cycle in yeast cultures attracted recently a significant interest. The Response/Signaling feedback models have been proposed to explain this phenomenon [2]. We apply a non-linear physiologically-structured PDE model of cell population to describe clusterinig in yeast cultures. The linear stability of stationary states will be discussed. We will present some results about clustering have been proven. In particular, in some cases the solutions of PDEc converge to traveling delta measures [1].

References

[1] Radosław Wieczorek, *PDE model of cell cycle dynamics and clustering in yeast*, submitted.

[2] Todd R. Young, Bastien Fernandez, Richard Buckalew, Gregory Moses, and Erik M. Boczko, *Clustering in cell cycle dynamics with general response/signaling feedback*, J. Theoret. Biol. **292** (2012), 103–115.

Agnieszka Bartłomiejczyk
Gdańsk University of Technology, Poland
Karolina Lademann
University of Gdańsk, Poland
Monika Wrzosek
University of Gdańsk, Poland

Newton's method for the McKendrick equation

We consider the initial value problem for the McKendrick equation with renewal

$$\begin{aligned} \frac{d\mathbf{u}}{dt} + \mathcal{A}(t)\mathbf{u} &= \mathbf{u}\,\Lambda(t,\mathbf{u}), \quad t \in [0,T], \\ \mathbf{u}(0) &= u_0 \end{aligned} \tag{1}$$

where

$$\begin{aligned} \mathcal{A}(t) &= c(t,x)\,\frac{\partial}{\partial x}, \\ \mathcal{D}(\mathcal{A}(t)) &= \{u(t,\cdot) \in C^1 \cap L^1 \cap L^\infty \;:\; u(t,0) = \mathcal{K}\mathbf{u}\}, \\ \mathcal{K}\mathbf{u} &= \int_0^\infty k(y)\,u(t,y)\,dy. \end{aligned}$$

We formulate the Newton scheme for (1) and prove its second order convergence.

Jerzy Zabczyk
Polish Academy of Sciences, Poland

Markovian models of short rates

In the theory of the bond market an important role play Markov processes R modelling the evolution of the short rate. They are nonnegative with the transition semigroup mapping the set of exponential functions $e^{-\lambda x-a}$, $\lambda, \gamma \geq 0$, into itslf. If, in addition, the process R has continuous trajectories, then it is a solution of the stochastic equation

$$dR(t) = (aR(t) + b)dt + \sqrt{R(t)}dW(t), \quad R(0) \geq 0.$$

In the equtiona, W is a Wiener process and a, b are constants, $b \geq 0$.

In the talk, based on a joint research with M. Barski, we describe known as well as some new results on the general case when the process R can have discontinuous trajectories. We will start from the discrete time situation.

References

M.Barski and J. Zabczyk *Bond Markets with Lévy factors*, book for CUP, in preparation

D. Filipović *A general characterization of one factor affine term structure models*, Finance and Stochastics, 5(2001), 389-412

D. Filipović and J. Zabczyk *Markovian term structure models in discrete time* Annals of Applied Probability, 12(2002), 710-729

F.Hubalek *A counterexample involving exponential-affine Laplace transforms*, unpublished, TU Vienna

K. Kawazu and S. Watanabe, *Branching processes with immigration and related topics*, Theory of Probability and its Applications, 16(1971), 36-54

Alyona A. Zamyshlyaeva
South Ural State University, Russia

Sobolev Type Equations of Higher Order. Theory and Applications

This report surveys the author's results concerning Sobolev type equations of higher order. The theory is constructed using the available facts on the solvability of initial (initial-final) problems for the first-order Sobolev type equations. The main idea is a generalization of the theory of degenerate (semi)groups of operators to the case of higher-order equations: decomposition of spaces and actions of the operators, construction of the propagators and the phase space for the homogeneous equation, as well as the set of valid initial values for the inhomogeneous equation. We use the phase space method, which is quite useful for solving the Sobolev type equations and consists in a reduction of a singular equation to a regular one defined on a certain subspace of the original space. As an application we reduce several mathematical models to initial (initial-final) problems for abstract Sobolev-type equations of higher order. The results may find further applications in the study of optimal control problems, nonlinear mathematical models, and in the construction of the theory of Sobolev-type equations of higher order in quasi-Banach spaces and stochastic spaces of noises.

Speakers