UNIVERSIDAD DE CÓRDOBA

A generalized notion for black holes using the causal boundary

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Outline

Black holes: Some basics

Causal boundaries: some ideas on the construction

Null infinity and Black holes

An application: Black holes in pp-waves

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How black holes are defined

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Regions on the spacetime where all (future) causal curves are trapped

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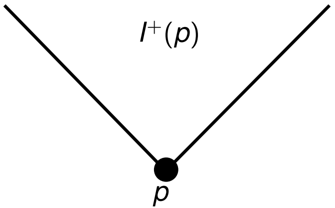

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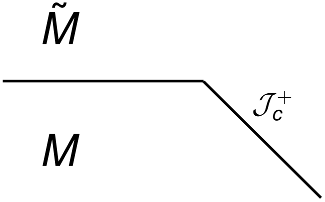
In fact, in the classical approach, the definition
of black hole makes use of the **conformal boundary**

Black Hole: Classical Definition

Consider $(M, g) \hookrightarrow (\tilde{M}, \tilde{g})$ a conformal embedding with conformal factor Ω.

➔ Define the notion of $\mathcal{J}_c^+$ on the conformal boundary.



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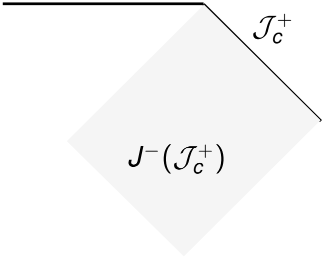

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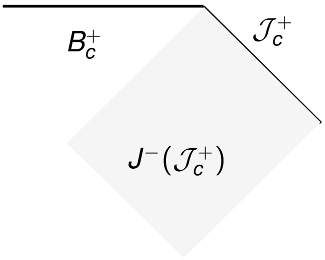
- ➔ Define the notion of $\mathcal{J}_c^+$ on the conformal boundary.
- ➔ Define the visible area as $J^-(\mathcal{J}_c^+)$.
- ➔ The black hole is defined as the complementary of previous set.



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However, not all models admit a conformal boundary

For instance pp-waves has no conformal boundary

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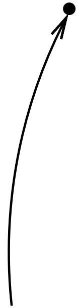

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Due to Geroch, Kronheimer and Penrose ('72) for strongly causal and time-oriented spacetimes:

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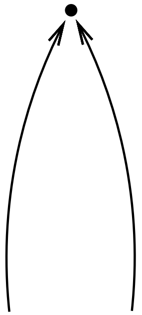

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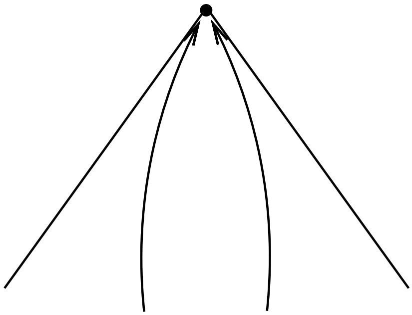

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$\hat{\partial} M$

Original Idea

➔ The same follows for the *past* causal boundary.

➔ Join together both boundaries accordingly...

But how?

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Terminal Indecomposable sets
if they are the past or future of a inextendible curve in M.

$P = I^-(\gamma)$ or $F = I^+(\eta)$.

Causal Boundary

P and F have to be S-related, $P \sim_S F$ which means

$$\begin{cases} P \text{ is a maximal IP in } \downarrow F := I^-\left(\{q \in V : q \ll p, \forall p \in F\}\right) \\ F \text{ is a maximal IF in } \uparrow P := I^+\left(\{p \in V : q \ll p, \forall q \in P\}\right) \end{cases}$$

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The boundary points $(P, F) \in \partial M$ are pairs of terminal sets

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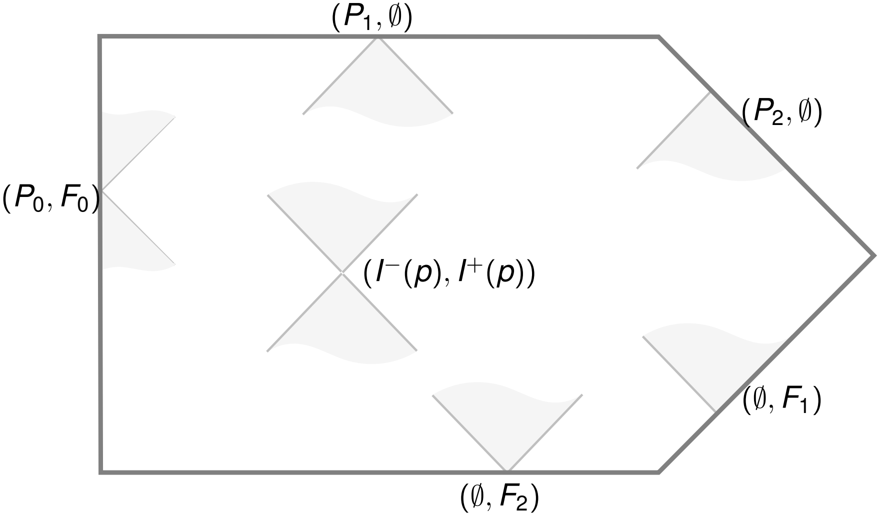

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Properties

The c-completion $\overline{M}$ satisfies:

(a) The causal structure and topology on M are preserved.

(b) The future and past of sets in $\overline{M}$ are open.

(c) Any timelike curve $\gamma \subset M$ has an endpoint in $\overline{M}$.

(d) It coincides with the conformal boundary under some mild hypothesis.

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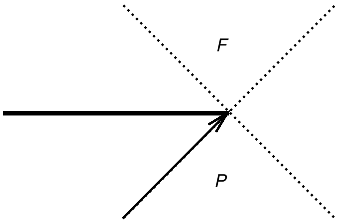

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...which follows if, for instance, (M, g) is
causally continuous

Outline

Null Infinity

Definition

The *future null infinity* of M, denoted by $\mathcal{J}^+$, is formed by pairs $(P, F) \in \partial M$ such that:

(I) $\exists$ a future complete **and future regular** null ray $\eta : [0, \infty) \to M$ with (P, F) as endpoint of η.

(II) every future-inextendible null geodesic with endpoint (P, F) is future complete.

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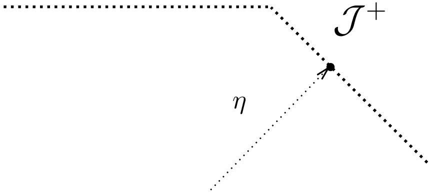
- ➔ (i) ensures that (P, F) is “far away”.
- ➔ Also *future regular* ensures a well behaviour between future and past sets.
- ➔ (ii) ensures that “there is no shortcut to infinity”.

Null Infinity, Visible points and Black Holes

Points that reach $\mathcal{J}^+$
are out of any black hole.



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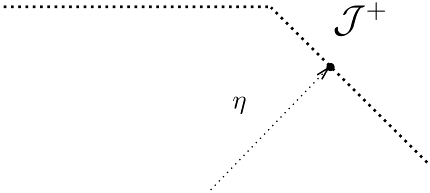

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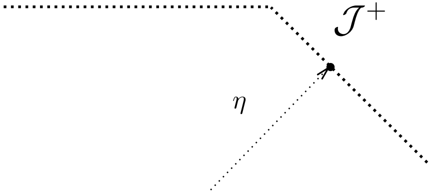

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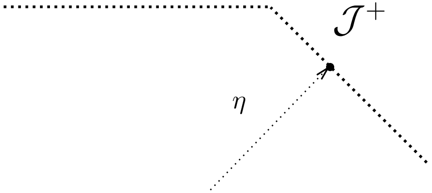

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And the *horizon* as $H^+ = \partial B^+$

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(b) $M = \mathrm{Int}(B^+) \cup H^+ \cup I^-(V_\infty)$.

- ➔ H^+ is an achronal and C^0 hypersurface.

(c) Any point in $H^+ \cap J^-(V_\infty)$ is connected with $\mathcal{J}^+$ with a null ray.

Some results on the non-existence of black holes

Proposition (Costa e Silva, Flores, -)

If M is globally hyperbolic, future null complete and with no compact Cauchy hypersurface, then $V_\infty \neq \emptyset$ but $B^+ = \emptyset$.

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- ➔ Such a curve has an endpoint in $\mathcal{J}^+$, and so, $p \in V_\infty$.

Some results on the non-existence of black holes

Theorem (Costa e Silva, Flores, -)

Suppose that M^{n+1} is a *strongly causal spacetime* with $n \geq 2$ satisfying:

(a) M is timelike and null geodesically complete,

(b) M satisfies the timelike convergence condition, $Ric(v, v) \geq 0$ for any timelike $v \in TM$,

(c) $\mathscr{J}^+ \neq \emptyset$.

(d) $\overline{M}$ is strongly properly causal.

Then $B^+ = \emptyset$.

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- From the hypothesis, p should be a *future trapped set*.
- Then, it should exist a causal line intersecting $E^+(p) = J^+(p) \setminus I^+(p)$.
- Such a curve has to be, in fact, timelike as $E^+(p)$ is contained in B^+.

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➔ Then, the Lorentzian splitting theorem ensures that M should be isometric to a product spacetime $\mathbb{R} \times S$, with complete Riemannian base.

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- Hence *M* is necessarily globally hyperbolic and, from previous result, *S* should be compact.

- But then, there is no achronal null geodesic, and so $\mathcal{J}^+ = \emptyset$, a contradiction.

Further results

In order to extend some of the classical results for black holes on this context, we require some regularity conditions

Prototype result

Assume that M is a Lorentz manifold with a ***regular*** null infinity $\mathcal{J}^+$, and let C be an achronal compact set. If C is not fully covered by a black hole, then there exists a future null C-ray with endpoint in $\mathcal{J}^+$.

Further results

In this sense, we need to consider two conditions:

- The null infinity $\mathscr{J}^+$ is ***ample*** if for any compact set $C \subset M$, and for any connected component $\mathscr{J}_0^+$ of $\mathscr{J}^+$, $\mathscr{J}_0^+ \cap (\overline{M} \setminus \widetilde{I^+(C)})$ is a non-empty open set, where

$$\widetilde{I^+(C)} := \{(P, F) \in \overline{M} : I^-(x) \subset P \text{ for some } x \in C\}.$$

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Remarks

- $\widetilde{I^+(C)}$ is closed if $\hat{M}$ is Hausdorff.
- It follows for some classical cases with a null conformal boundary with past-complete null geodesic generators.

Further results

- The null infinity $\mathcal{J}^+$ is ***past-complete*** if given $(P, F) \in \mathcal{J}^+$, any $(P', F') \in \partial M$ with $P' = I^-(\eta)$, being η a future-directed inextendible null geodesic generator of ∂P, also belong to $\mathcal{J}^+$.

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Definition

We will say that $\mathcal{J}^+$ is ***regular*** if it is both ample and past complete.

Further results

Theorem (Costa e Silva, Flores, -)

Assume that M is a Lorentz manifold with a ***regular*** null infinity $\mathcal{J}^+$, and let C be an achronal compact set. If C is not fully covered by a black hole, then there exists a future null C-ray with endpoint in $\mathcal{J}^+$.

Further results

Theorem (Costa e Silva, Flores, -)

Assume that M is a Lorentz manifold with a ***regular*** null infinity $\mathcal{J}^+$, and let C be an achronal compact set. If C is not fully covered by a black hole, then there exists a future null C-ray with endpoint in $\mathcal{J}^+$.

Corollary

Assume that $\mathcal{J}^+$ is regular and that the null convergence condition holds in (M, g). If $S \subset M$ is a closed trapped surface, then $S \subset B^+$.

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Generalized plane waves

Consider a *Generalized Plane Wave*

$$M = M_0 + \mathbb{R}^2, \quad g \equiv g_0 + 2dudv + H(x,u)du^2$$

$$(M_0, g_0) \text{ Riemannian}, \qquad H : M_0 \times \mathbb{R} \to \mathbb{R}$$

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Remarks

- ⋆ Its causal boundary is known.
- ⋆ Under the assumption that the null rays $\gamma_{x,u}(s) = (x, -s, u)$ are future-regular ($\uparrow \gamma = \uparrow I^-(\gamma)$), we compute (at least, partially) $\mathcal{J}^+$.

Generalized plane waves

Theorem (Costa e Silva, Flores, -)

If M is a geodesically complete generalized plane wave whose null rays $\gamma_{x,u}$ are future-regular, then it does not contain black holes.

This is a formalization of a previous result given first by Hubeny-Randamani, and later generalized by Flores-Sanchez.

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Corollary

If M is a geodesically complete causally continuous generalized plane wave, then it does not contain black holes.

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Thanks for your attention