Local Invariants of Planar Caustics

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The invariants in the title are invariants of Lagrangian maps from surfaces to $\mathbb{R}^2$ whose increments in generic homotopies are determined entirely by diffeomorphism types of local bifurcations of the caustics. Such invariants are dual to trivial codimension 1 cycles supported on the discriminant in the space $\mathcal{L}$ of the Lagrangian maps.

We obtain a description of the spaces of the discriminantal cycles (possibly non-trivial) for the Lagrangian maps of an arbitrary closed surface, both for the integer and mod2 coefficients. We show that all integer local invariants of caustics of Lagrangian maps without corank 2 points are essentially exhausted by the numbers of various singular points of the caustics and the Ohmoto-Aicardi linking invariant of ordinary maps. As an application, we use the discriminantal cycles to establish non-contractibility of certain loops in $\mathcal{L}$.