THE CLOSEDNESS THEOREM OVER HENSELIAN VALUED FIELDS AND ITS APPLICATIONS

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The aim of this talk is to develop geometry of algebraic subvarieties of K^n over arbitrary Henselian valued fields K of equicharacteristic zero. This is a continuation of my previous article, devoted to algebraic geometry over rank one valued fields, which in general requires more involved techniques and to some extent new treatment. Again, at the center of my approach is the closedness theorem that the projections $K^n \times \mathbb{P}^m(K) \to K^n$ are definably closed maps. Hence we obtain a descent property for blow-ups, which enables applications of resolution of singularities in much the same way as over the locally compact ground field. As before, the proof of that theorem uses i.a. certain cell decomposition, the local behaviour of definable functions of one variable and fiber shrinking, a relaxed version of curve selection. But now, to achieve the former result, I first examine functions given by algebraic power series. The results established include: several versions of curve selection and of the Łojasiewicz inequality, piecewise continuity of definable functions and Hölder continuity of functions on closed bounded subsets of K^n, extending continuous hereditarily rational functions and the theory of regulous functions, sets and sheaves. Two basic tools applied are quantifier elimination for Henselian valued fields due to Pas and relative quantifier elimination for ordered abelian groups (in a many-sorted language with imaginary auxiliary sorts) due to Cluckers–Halupczok.