School of Mathematics and Statistics
University College Dublin
Belfield, Dublin 4, Ireland

June 29, 2022

Scientific Council of the Institute of Mathematics
Polish Academy of Sciences, Warsaw

Report on the PhD thesis of Lukasz Maślanka:

This is an outstanding thesis which presents several new results concerning asymptotic properties of the Robinson-Schensted correspondence. It is based on three papers co-authored with the candidates PhD supervisor Professor Śniady, two of which also include M. Marciniak (another of Professor Śniady's PhD students) as a co-author. One of these papers has already been published in *Probability Theory and Related Fields*, which is among the the highest ranked journals in probability.

The subject area of the thesis falls in the domain of *asymptotic representation theory*, a subject which was initiated by the seminal works of Kerov, Vershik and others in the 1980's, most famously in the context of the celebrated longest increasing subsequence problem. The area has received considerable attention and development in recent years, not least by the candidates PhD supervisor, owing to its many deep connections with random matrices, free probability, combinatorics and interacting particle systems. This thesis builds on several recent works of Professor Śniady and his collaborators.

The first chapter provides some useful background and motivation for the topics considered in the thesis. It is very well presented and very clear.

The second chapter considers the Robinson-Schensted algorithm with random input. The main result of this chapter is a (functional) Poisson limit theorem for the growth of the bottom few rows. It shows that, in an appropriate scaling, after a long time, the bottom rows evolve like independent Poisson counting processes. It is explained how this may be interpreted as a statement about the long-time behaviour of the multi-line Hammersley process of Ferrari and Martin (2009), and includes a classical result of Aldous and Diaconis (1995) in the case of one row. The proof is highly non-trivial and requires a deep understanding of the works of Hammersley (1972), Vershik and Kerov (1977) and Logan and Shepp (1977) on the longest increasing subsequence problem. In particular, it is obtained by giving a precise formulation and proof of an assumption of independence (between rows) which was previously proposed (imprecisely) by Hammersley and had been left open in the literature. The proof of this is obtained by a clever adaptation of the proof of a lemma which appeared in the work of Vershik and Kerov. The chapter also includes some interesting conjectures on how the results may be generalised in the context of Schur-Weyl insertion tableaux, and various scaling limits within that framework.

In Chapter 3, the focus shifts to bumping routes at the edge of the insertion tableau. It is proved that, in an appropriate coordinate system, the bumping route converges in distribution to the Poisson process. This significantly adds to earlier pioneering work of Romik and Śniady (2016), which considered finite bumping routes in a natural scaling. Here the authors consider infinite bumping routes in a different scaling which focuses attention on the edge of the insertion tableau. This is a substantial undertaking, with many technical hurdles to overcome, and the result is both illuminating and constitutes significant progress in the field, as evidenced by its publication in *Probab. Theory Relat. Fields.*

Chapter 4 is devoted to the asymptotic study of typical evacuation paths and sliding paths in Young tableaux of square, and more general, shape. A nice interpretation of the latter in terms of the trajectory of a second class particle in a TASEP-like particle system with prescribed initial and ending positions, equipped with the uniform measure on all possible histories. This problem is further motivated by similar questions which have been studied by Angel and others on random sorting networks and related topics. In any case, it is a natural and interesting question in its own right, and is addressed in this chapter via some deep connections to the representation theory of the symmetric group. The main results give the precise asymptotic behaviour of typical evacuation paths and sliding paths in large Young tableau of a given (asymptotic) shape. The proofs use some new and important ideas, specifically the notions of 'surfers' and 'multisurfers', and require substantial asymptotic analysis. They also use in a crucial way, known results, due to Féray and Śniady (2011) and Biane (1998, 2001), on the asymptotics of symmetric group characters and approximate factorisation property. In all, this chapter represents a very substantial and important piece of work, which also brings together many different aspects of the subject in a beautiful way.

This is an outstanding thesis which would certainly merit the award of PhD, in my opinion and experience, at any leading university. As such, I would strongly recommend that the Council support the motion to confer the degree of doctor of mathematical sciences upon MSc Łukasz Maślanka.

Yours sincerely,

Neil O'Connell