Report on Jacek Krajczok's PhD thesis: Modular properties of locally compact quantum groups

The thesis is concerned with the study of locally compact quantum groups in the operator algebraic setting. The thesis is 138 pages long and consists of 6 sections and an appendix. The first two sections cover basics of the theory of locally compact quantum groups and introduce a few standard examples that play a prominent role later. The appendix contains a discussion of direct integrals and a few technical lemmas from operator theory and quantum groups.

Section 3 is based on Krajczok's paper "Modular properties of type I locally compact quantum groups", to appear in J. Operator Theory. A Plancherel type formula for quantum groups of type I was obtained by Desmedt and his results were further refined by Caspers and Koelink. Krajczok completes their analysis of modular properties of such quantum groups by giving explicit formulas for the modular operators and the modular element in terms of the Duflo–Moore operators. As examples he considers the discrete dual of $SU_q(2)$ and the quantum $az + b$ group.

Section 4 is based on the joint paper with Sołtan "The quantum disk is not a quantum group", to appear in J. Topol. Anal. The main result is the statement in the title - the quantum disk, that is, the Toeplitz algebra, does not admit the structure of a compact quantum group. The proof relies on results from the previous section. The strategy is to show using those results that the hypothetical quantum group must be of Kac type, while it is easy to see that this is not possible by looking at the tracial states of the Toeplitz algebra.

Section 5 is based on the joint preprint with Wasilewski "On the von Neumann algebra of class functions on a compact quantum group". In my opinion this is the most interesting part of the thesis. It is motivated by the question whether the characters of a free orthogonal quantum group of non-Kac type generate a maximal abelian subalgebra (masa) of the function algebra. The authors' approach, eventually leading to the negative answer for a large class of non-Kac quantum groups, is inspired by work of Bikram and Mukherjee, who used the theory of quasi-split inclusions of von Neumann algebras to study masas in q-deformed Araki–Woods von Neumann algebras.

Section 6 studies the question whether for discrete quantum groups amenability is equivalent to w*-completely positive approximation property (w*-CPAP) of their von Neumann algebras. Krajczok obtains a partial answer showing that amenability is equivalent to a relative version of w*-CPAP.

Jacek Krajczok has written a very interesting thesis and demonstrated that he masters a variety of tools for studying von Neumann algebras and quantum groups. The presentation is clear, with a careful attention to technical details. In my opinion, the thesis fulfils the requirements for awarding a PhD degree in mathematics.

March 31, 2022  Sergey Neshveyev